USE OF DIMENSIONLESS NUMBERS IN ANALYZING MELT FLOW AND MELT COOLING PROCESSES

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ABSTRACT

Dimensionless analysis is a powerful tool in analyzing the transient heat transfer and flow processes accompanying melt flow in an injection mold or cooling in blown film,to quote a couple of examples. However, because of the nature of non-Newtonian polymer melt flow the dimensionless numbers used to describe flow and heat transfer processes of Newtonian fluids have to be modified for polymer melts. This paper describes how an easily applicable equation for the cooling of melt in a spiral flow in injection molds has been derived on the basis of modified dimensionless numbers and verified by experiments. Analyzing the air gap dynamics in extrusion coating is another application of dimensional analysis.

PREDICTING FLOW LENGTH IN INJECTION MOLDS

Injection molding is widely used to make articles out of plastics for various applications. One of the criteria for the selection of the resin to make a given part is whether the melt is an easy flowing type or whether it exhibits a significantly viscous behavior. To determine the flowability of the polymer melt the spiral test, which consists of injecting the melt into a spiral shaped mold shown in Figure 1, is used. The length of the spiral serves as a measure of the ease of flow of the melt in the mold, and enables mold and part design suited to material flow.
The experimental flow curves obtained at constant injection pressure under given melt temperature, mold temperature and axial screw speed are given schematically in Figure 2 for a resin type at various spiral heights with melt flow index of the polymer brand as parameter. By comparing the flow lengths with one another at any spiral height also called wall thickness, the flowability of the resin brand in question with reference to another brand can be inferred (1), (2).

MODEL

The transient heat transfer and flow processes accompanying melt flow in an injection mold can be analyzed by state of the art commercial software packages. However, for simple mold geometries such as the one used in the spiral test it is possible to predict the melt flow behavior on the basis of dimensionless numbers and obtain formulae useful for the practice. These relationships can be easily calculated with a handheld calculator offering quick estimates of the target values. Due to the nature of non-Newtonian flow the dimensionless numbers used to describe flow and heat transfer processes of Newtonian fluids have to be modified for polymer melts. According to the authors (3), (4) the movement of a melt front in a rectangular cavity can be correlated by Graetz number, Reynolds number, Prandtl number and Brinkman number. As the flow length in a spiral test depends significantly on the injection pressure (Figure 4) the Euler number (5) is included in the present work in order to take the effect of injection pressure on the flow length.
The Graetz number Gz based on the flow length is given by

$$\mathrm{Gz} = \frac{G \cdot c_p}{\lambda \cdot L} \tag{1}$$

with G melt throughput, c_p specific heat, λ thermal conductivity of the melt and L spiral length in the spiral test.
The mean velocity of the melt front V_e in the cavity is defined by

$$V_e = \frac{Q}{A} \quad \text{with} \quad Q = \frac{G}{\rho} \tag{2}$$

where Q is the volume throughput, A the area of the spiral cross section (Figure 1) and ρ the density of the melt. The area A follows from

$$A = W \cdot H + H^2 \cdot \tan\alpha \tag{3}$$

with W the base width of the spiral and H spiral height or wall thickness as shown in Figure 1.

MELT VISCOSITY AND POWER LAW EXPONENT

Melt viscosity and power law exponent can be calculated by means of the rheological models according to Carreau, Muenstedt, Klein and Ostwald and de Waele as shown by Rao (6). These values can be directly obtained from the database which evaluates the measured viscosity data, and stores the coefficients occurring in these models together with thermal properties in the data bank (6). The reciprocal value of power law exponent according to Muenstedt model (7), for example, is given by

$$n_r = 1 + A_1 + 2 \cdot A_2 \cdot \lg(a_T \cdot \dot{\gamma}) + 3 \cdot A_3 \cdot (\lg(a_T \cdot \dot{\gamma}))^2 + 4 \cdot A_4 \cdot (\lg(a_T \cdot \dot{\gamma}))^3 \tag{4}$$

The melt viscosity η_a is calculated from

$$\eta_a = \lg a_T + A_0 + A_1 \cdot \lg(a_T \cdot \dot{\gamma}) + A_2 \cdot (\lg(a_T \cdot \dot{\gamma}))^2 + + A_3 \cdot (\lg(a_T \cdot \dot{\gamma}))^3 + A_4 \cdot (\lg(a_T \cdot \dot{\gamma})) \tag{5}$$

The shear rate $\dot{\gamma}$ in Eq.(4) and Eq (5) is obtained from

$$\dot{\gamma} = \frac{6 \cdot Q}{W_{mean} \cdot H^2} \quad \text{where } W_{mean} = W + H \cdot \tan\alpha . \tag{6}$$

A_0, A_1, A_2, A_3, A_4 are material constants, and a_T the shift factor. For amorphous polymers the shift factor is obtained from (8)

$$\lg a_T = \frac{-c_1 \cdot (T - T_0)}{c_2 + (T - T_0)} \tag{7}$$

with constants c_1, c_2 and melt and reference temperatures T and T_0, respectively, in K. For semi-crystalline and crystalline polymers a_T is calculated from

$$a_T = b_1 \cdot e^{b_2 / T} \tag{8}$$

with the constants b_1, b_2 and melt temperature T in K.

The following dimensionless numbers are obtained from (3), (5), (9):

$$\textit{Reynolds number } \mathrm{Re} = \frac{V_e^{(2-n_r)} \cdot \rho \cdot H^{*n_r}}{k^*} \tag{9}$$

with $H^* = 0.5 \cdot H$ and $k^* = \eta_a \cdot \dot{\gamma}^{(1-n_r)}$

$$\textit{Prandtl number } \mathrm{Pr} = \frac{k^* \cdot c_p \cdot H^{*(1-n_r)}}{\lambda \cdot V_e^{(1-n_r)}} \tag{10}$$

$$\text{Brinkman number } \mathrm{Br} = \frac{k^* \cdot V_e^{(1+n_r)} \cdot H^{*(1-n_r)}}{\lambda \cdot (T_M - T_W)} \tag{11}$$

$$\textit{Euler Number } \mathrm{Eu} = \frac{100 \cdot p_I}{\rho \cdot V_e^2} \tag{12}$$

where p_I is the injection pressure.

EXPERIMENTAL RESULTS AND DISCUSSION

The experimental flow curves for four different resins measured at constant injection pressure under different processing conditions and spiral wall thicknesses are given in Figure 3. The flow length as a function of injection pressure is shown in Figure 4 for LDPE as an example. The Graetz numbers calculated from the experimentally determined spiral lengths at different operating conditions and resins are plotted as function of the product $Re \cdot Pr \cdot Br \cdot Eu$ as shown in Figure 5. As can be seen from this Figure the correlation of the Graetz number with this product is good and thus for any particular material the spiral length can be predicted from the relationship

$$Gz = f\,(Re \cdot Pr \cdot Br \cdot Eu) \qquad (13)$$

Figure 6 shows the good agreement between measured and calculated spiral lengths for the experimentally investigated resins.

SAMPLE CALCULATION

The example given below shows as to how the flow length of a given resin can be calculated from Eq. (13). For the input values W = 10 mm, H = 2 mm, = 1.06 g/cm^3, c_p = 2 KJ/kg K, = 1.5 W/K m, T_M = 270°C, T_W = 70°C and G = 211.5 kg/h, as well as A_0 = 4.7649, A_1 = -0.4743, A_2 = -0.2338, A_3 = 0.081, A_4 = -0.01063, c_1 = 4.45, c_2 = 146.3 and T_0 = 190°C following output is obtained: Re = 0.05964, Pr = 76625.34, Br = 1.7419 and Eu = 10825.84. The Graetz number Gz for the product Re Pr Br Eu follows from Figure 5: Gz = 217.63. Hence L = 420 mm.

AIR GAP DYNAMICS IN EXTRUSION COATING

The cooling of the film in the air gap between the exit of an extrusion flat die and the nip where the web touches the roll has a significant effect on the adhesion of the laminates. This cooling process can be conveniently described with the help of the Graetz number, Eq. (1), as shown in the paper [10].

CONCLUSIONS

The spiral test is used for classifying injection molding resins with respect to their melt flow behavior. By applying dimensional analysis to measured flow curves of thermoplastic resins a relationship was obtained to predict the flow length as a function of melt temperature, mold temperature, injection speed, injection pressure and spiral geometry.
By means of this practical equation which can be easily calculated by a hand-held programmable calculator experimentation can be minimized and flow lengths estimated for thermoplastic resins. A sample calculation is given to illustrate the use of the equation to calculate the flow length.
The application of dimensional analysis for analyzing the cooling in the air gap in extrusion coating processes is mentioned.

ACKNOWLEDGEMENT

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Figure 1: Schematic representation of spiral form

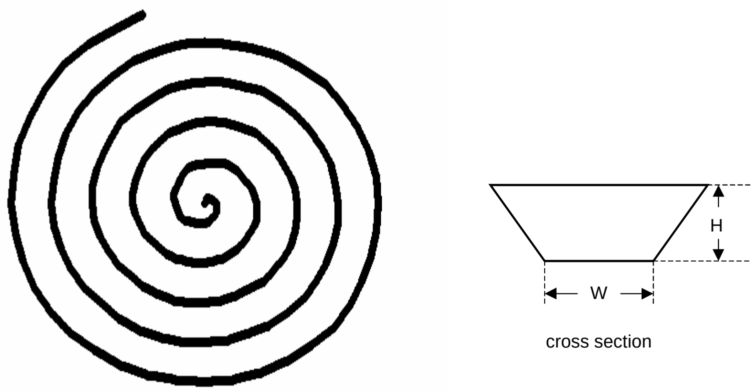


Figure 2: Schematic flow curves

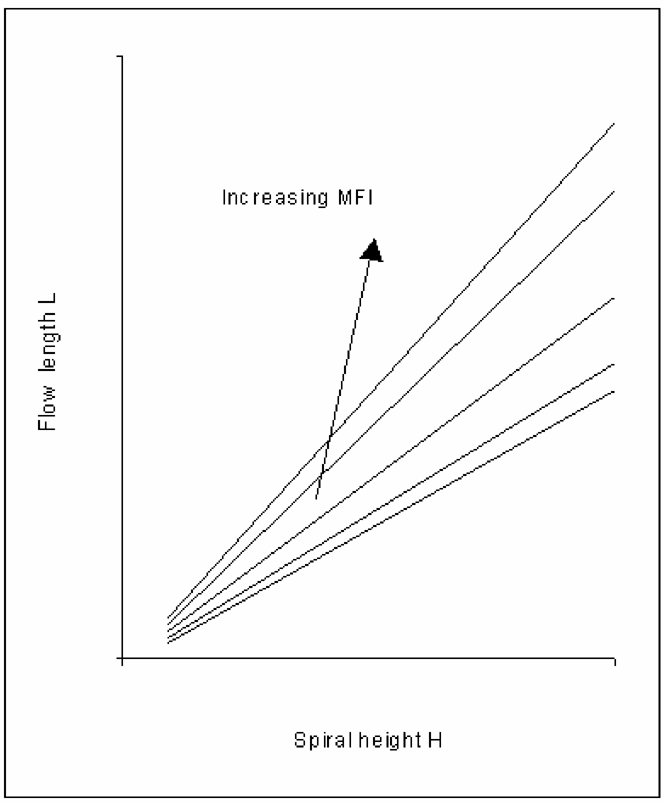


Figure 3: Experimental flow curves for LDPE, HDPE, PP and PS

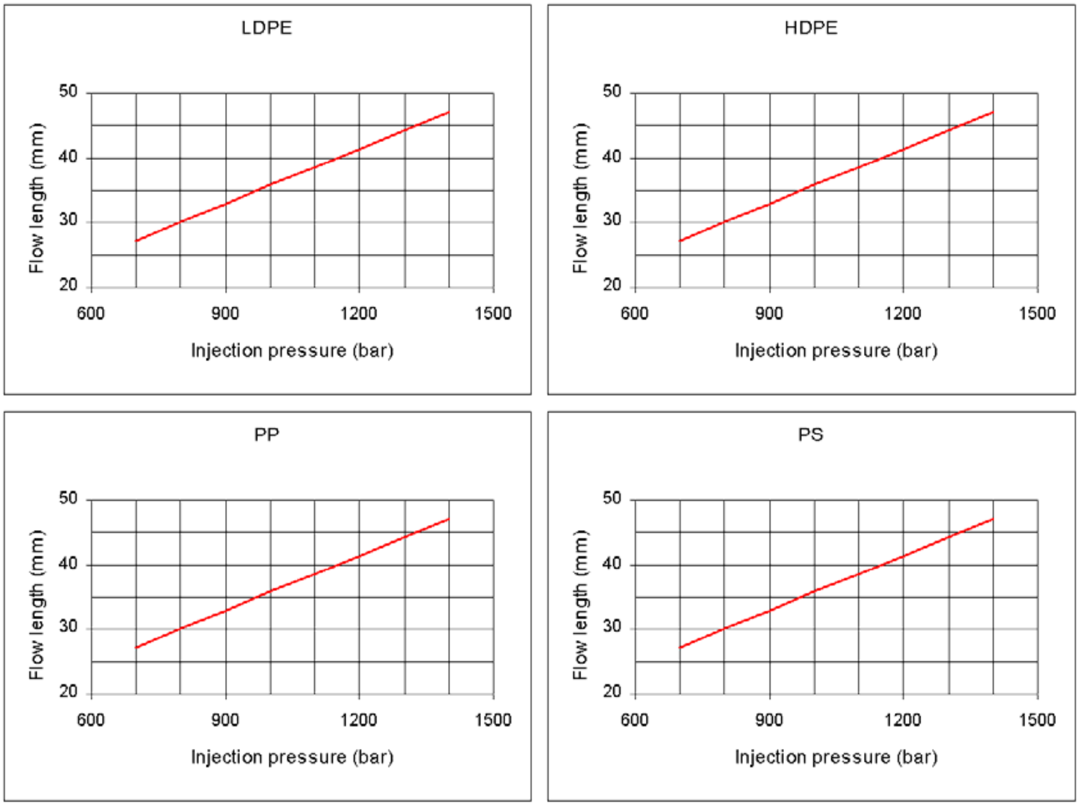


Figure 4: Effect of injection pressure on flow length

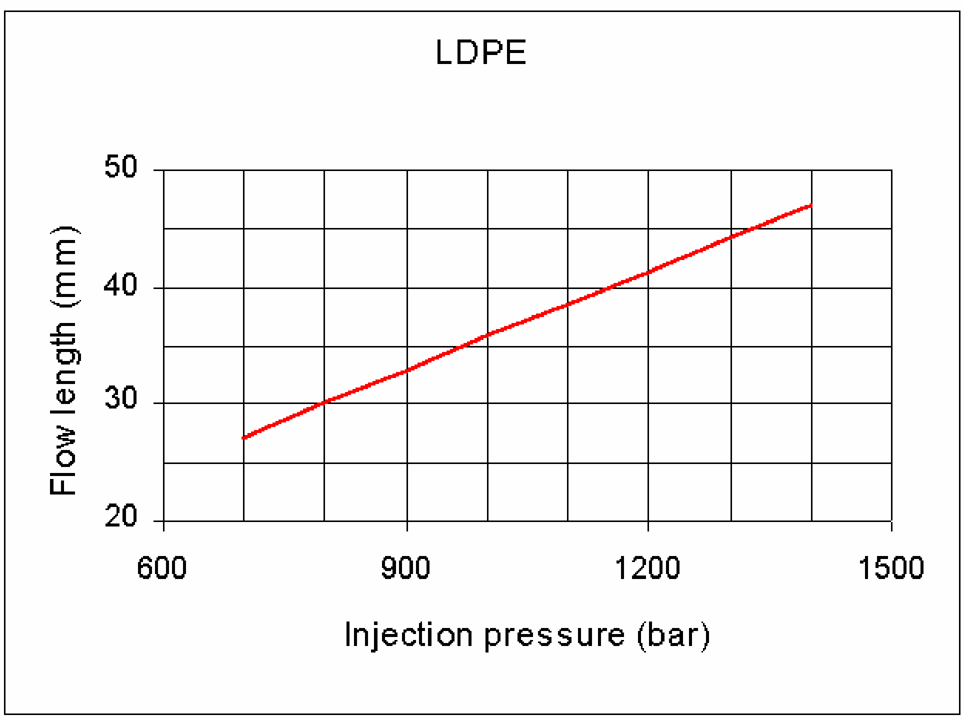


Figure 5: Graetz number as a function of product Re Pr Br Eu for different resins

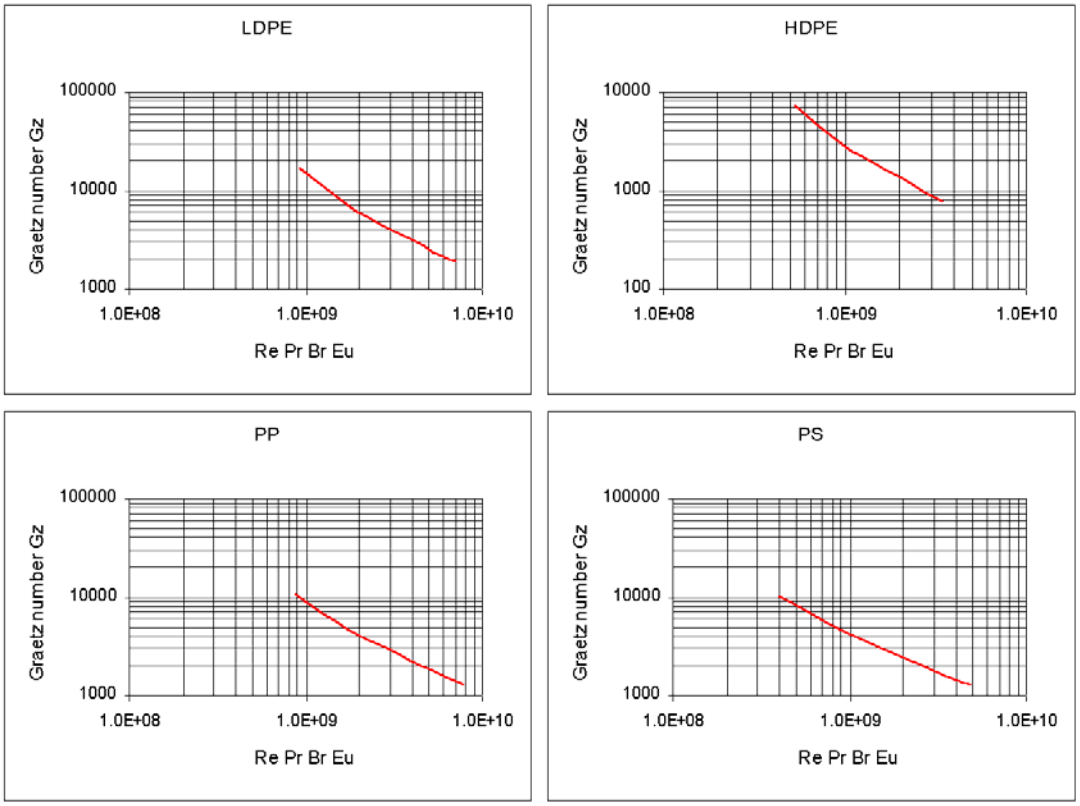
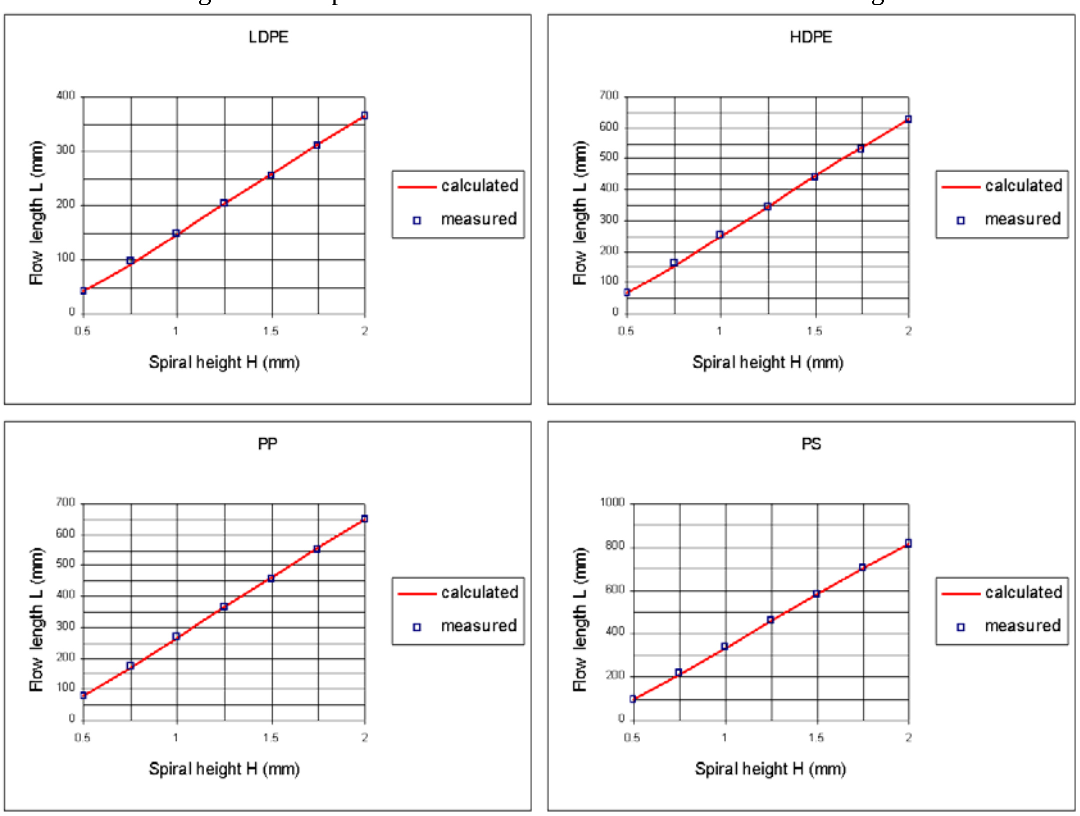


Figure 6: Comparison between measured and calculated flow length



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USE OF DIMENSIONLESS NUMBERS IN ANALYZING MELT FLOW AND MELT COOLING PROCESSES

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Use of Dimensionless Numbers in Analyzing Melt Flow and Melt Cooling Processes

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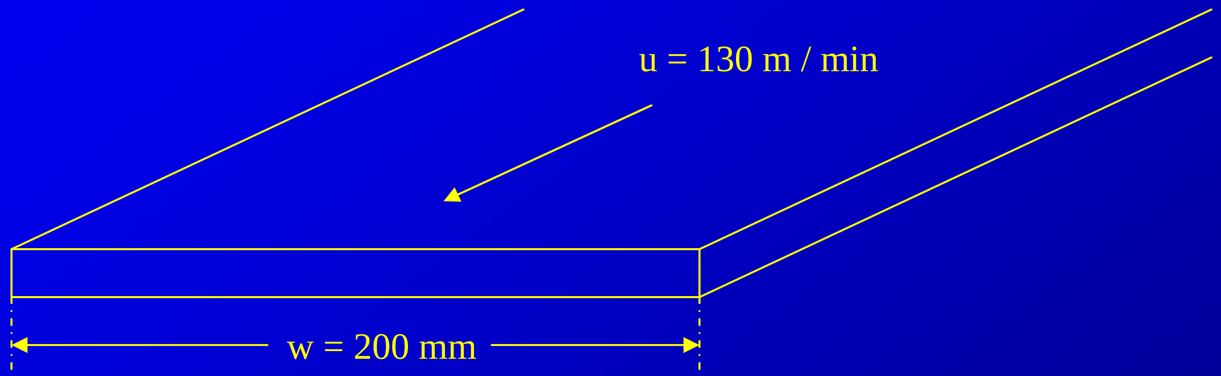
Example

Reynolds number $\mathrm{Re} = \rho \cdot u \cdot l / \eta$



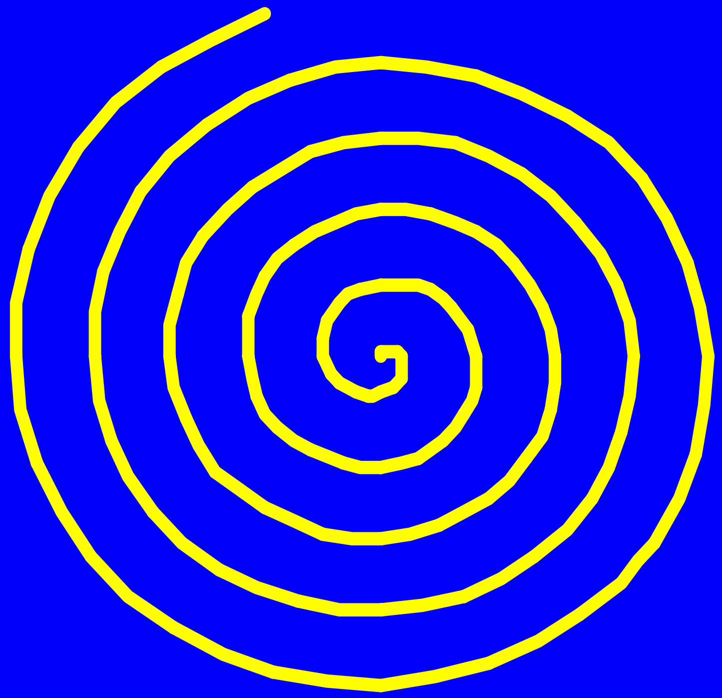
$$\mathrm{Re} = u \cdot w / \nu = \frac{130}{60} \cdot 0.2 / 17.86 \cdot 10^{-6} = 24262$$

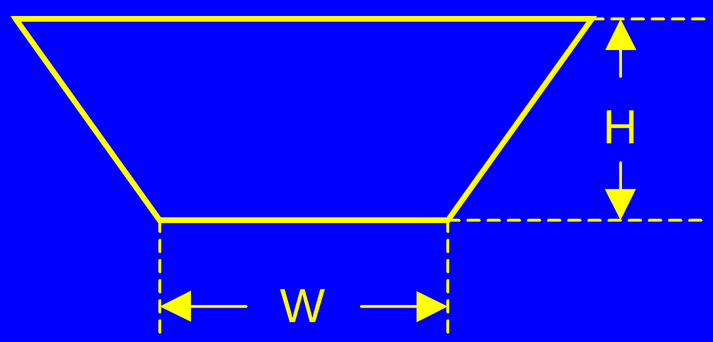
$$\alpha = f(\mathrm{Re})$$

What is the advantage
of using
dimensionless numbers?

Dimensionless groups can be used to describe complicated processes which are influenced by a number of variables.
The whole process can be analyzed on a sound basis by means of a few dimensionless parameters.

Schematic representation of sprial form



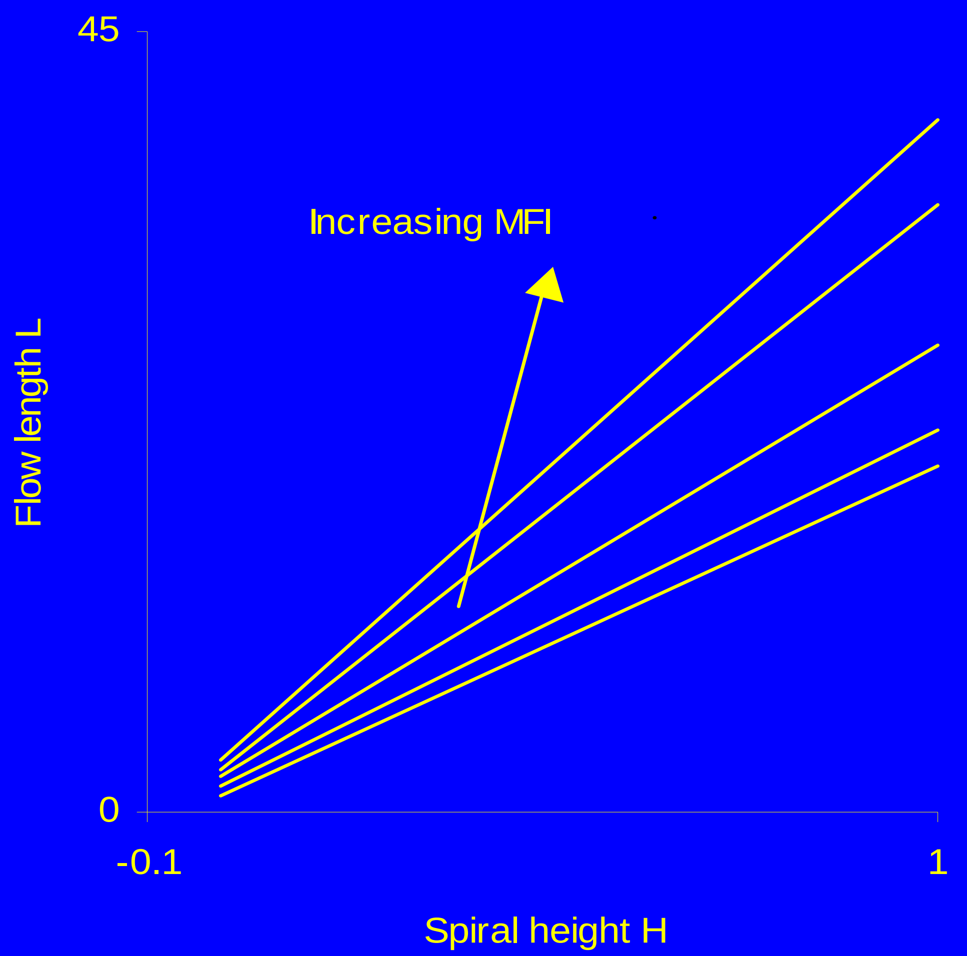


cross section

Aim

- To develop a practical relationship for predicting the spiral flow length in order to be able to reduce experimentation.

Schematic flow curves



Parameters influencing flow length

- Spiral height and width
- Melt temperature
- Mold temperature
- Injection pressure
- Travel velocity of the injection unit

Dimensionless numbers used

- Reynolds number
- Prandtl number
- Brinkman number
- Euler number
- Graetz number

Reynolds number: $$\mathrm{Re} = \frac{V_e^{(2-n_r)} \cdot \rho \cdot H^{*n_r}}{k^*}$$

Prandtl number: $$\mathrm{Pr} = \frac{k^* \cdot c_p \cdot H^{*(1-n_r)}}{\lambda \cdot V_e^{(1-n_r)}}$$

Brinkman number: $$\mathrm{Br} = \frac{k^* \cdot V_e^{(1+n_r)} \cdot H^{*(1-n_r)}}{\lambda \cdot (T_M - T_W)}$$

Euler number: $$\mathrm{Eu} = \frac{100 \cdot p_I}{\rho \cdot V_e^2}$$

$$H^* = 0.5 \cdot H \qquad k^* = \eta_a \cdot \dot{\gamma}^{(1-n_r)}$$

Graetz number:

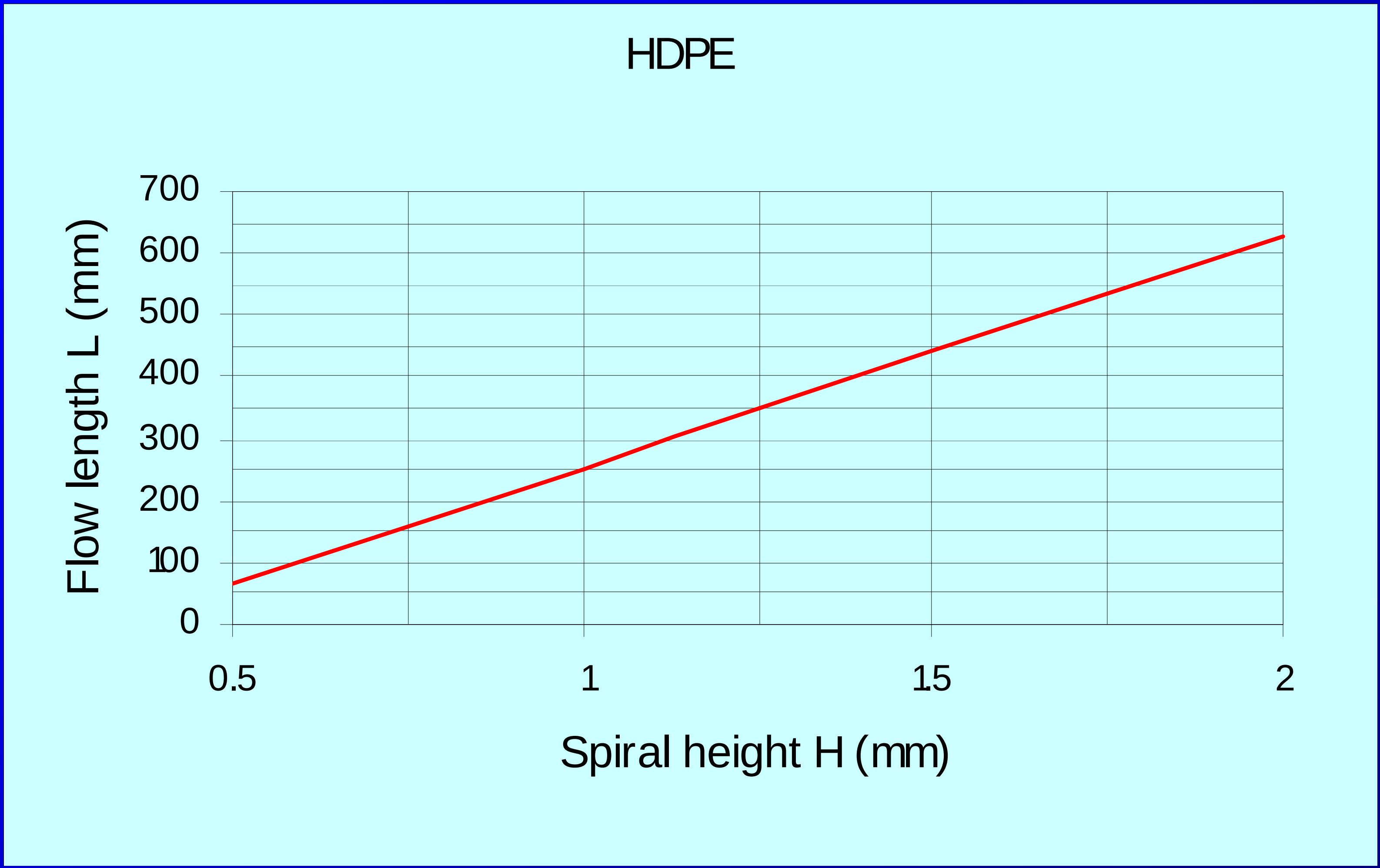
$$\mathrm{Gz} = \frac{G \cdot c_p}{\lambda \cdot L}$$

Relationship derived from the experimental values

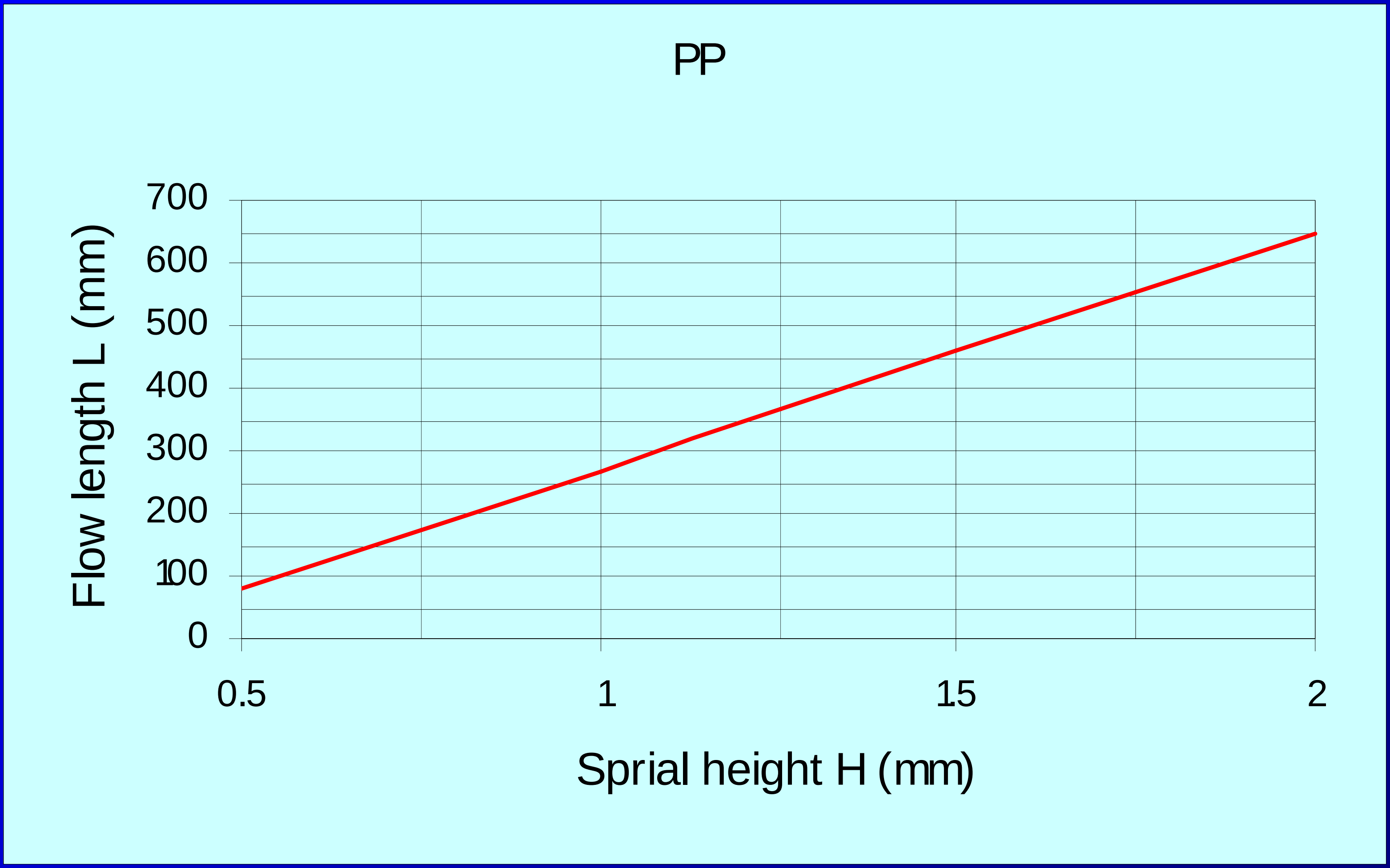
$$Gz = f(\mathrm{Re} \cdot \mathrm{Pr} \cdot \mathrm{Br} \cdot \mathrm{Eu})$$

Experimental flow curves

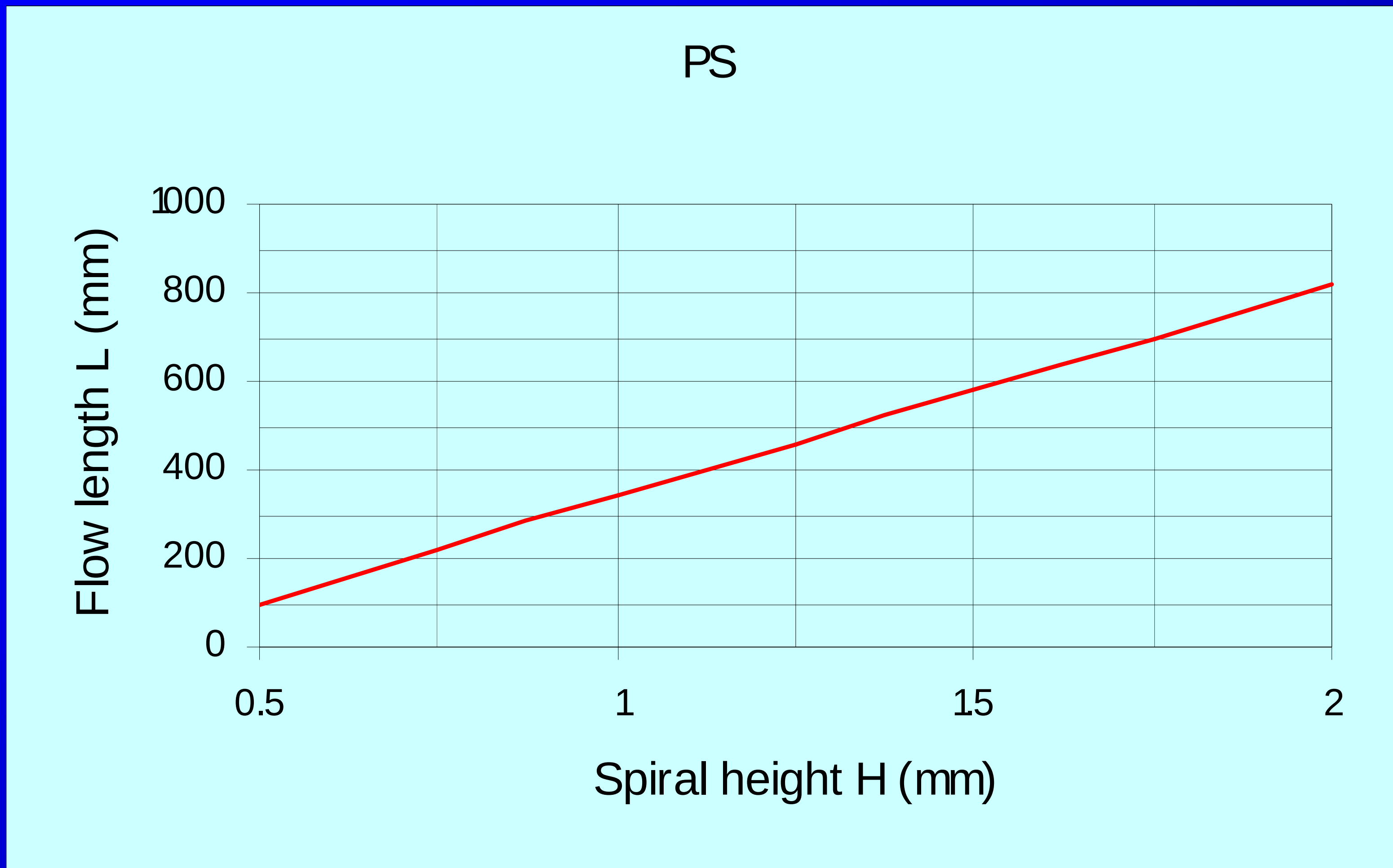
Experimental flow curves



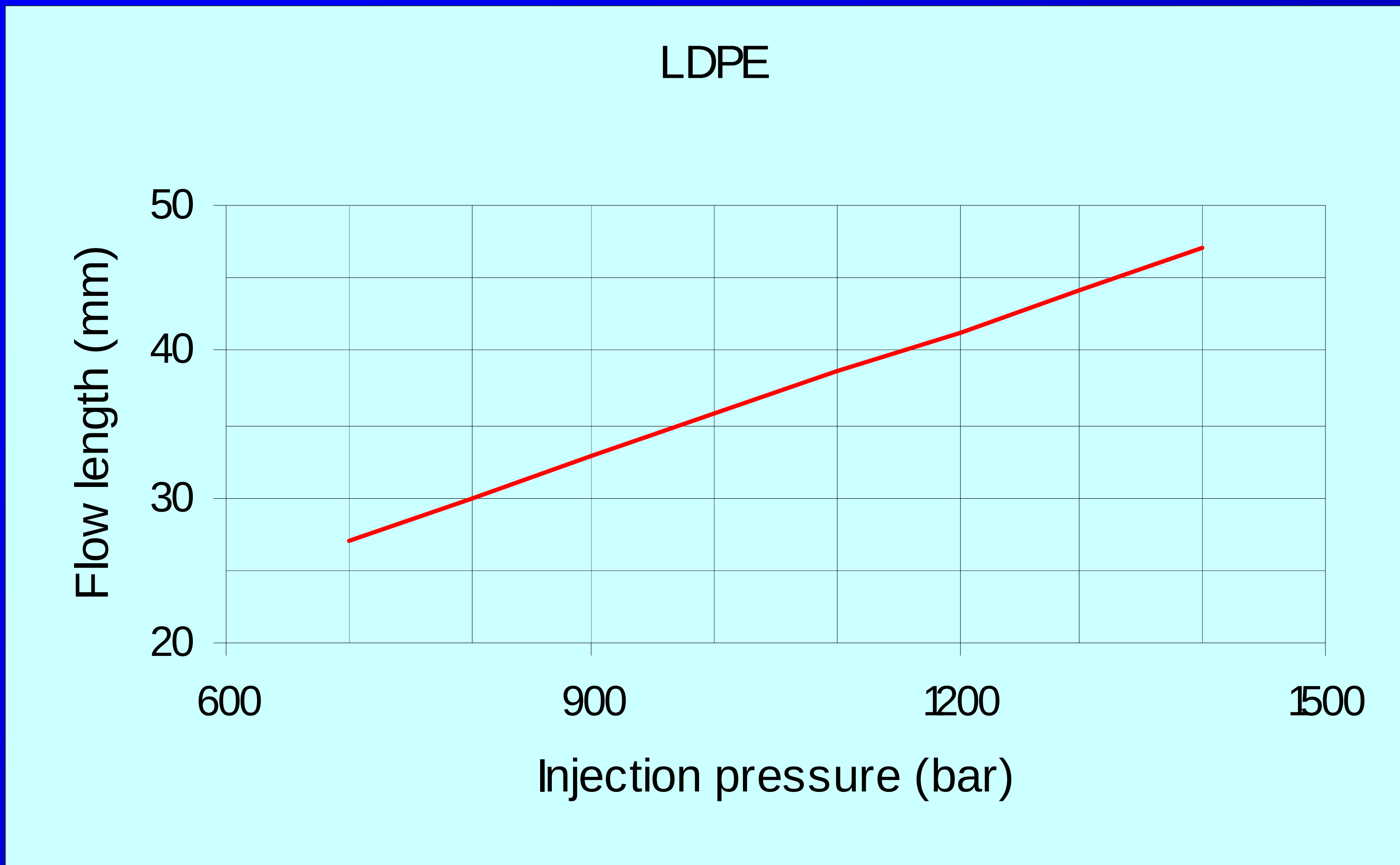
Experimental flow curves



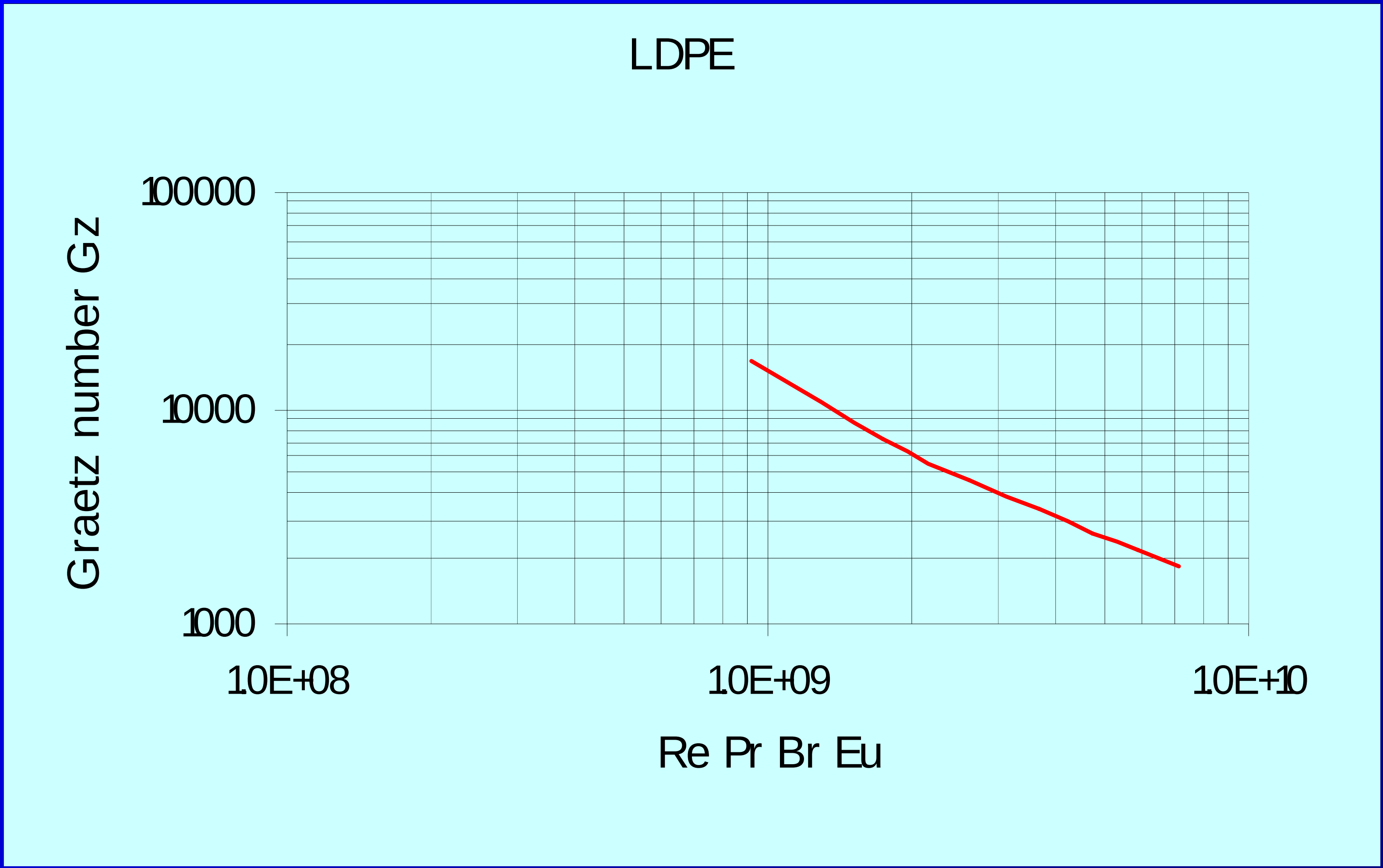
Experimental flow curves



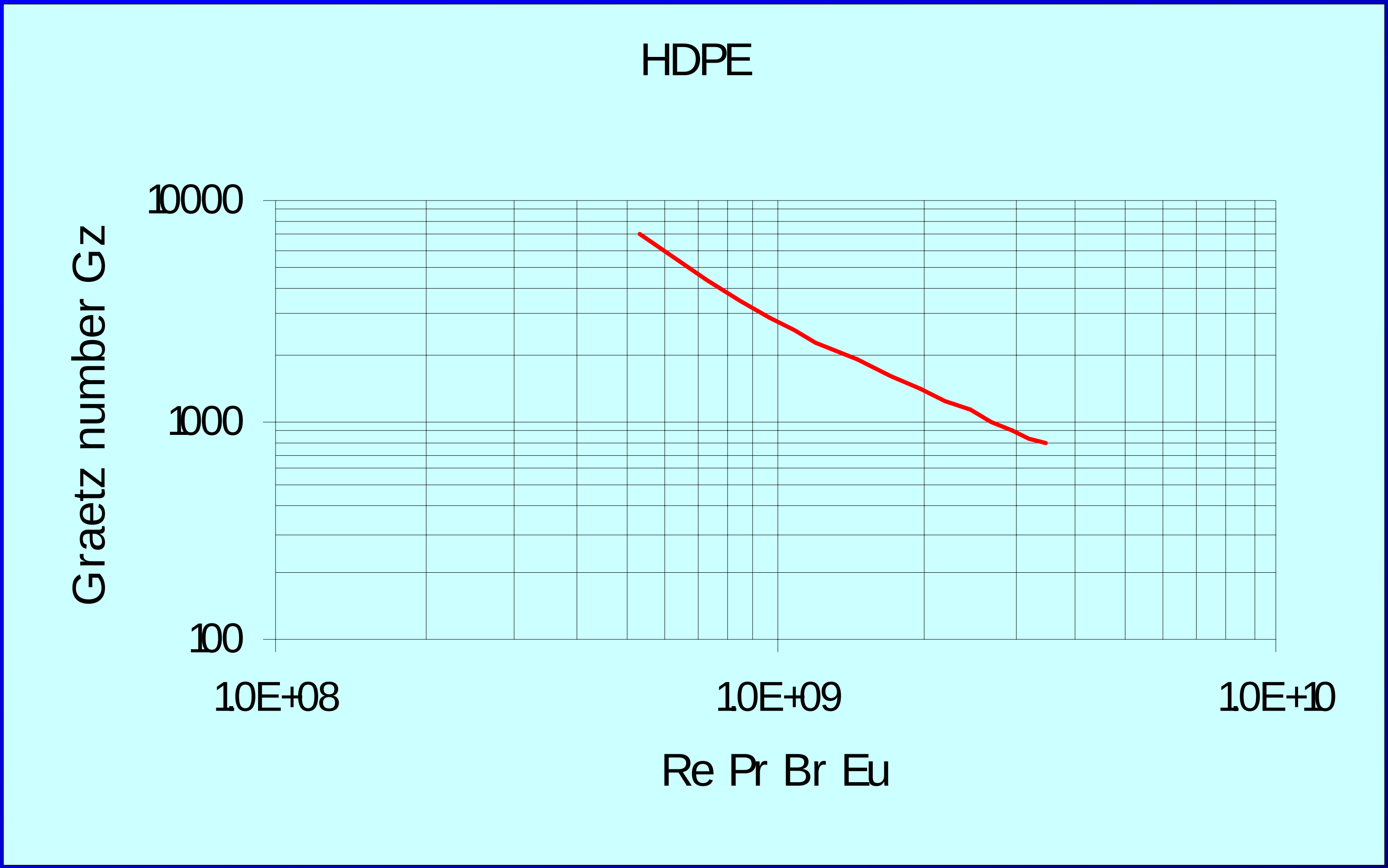
Effect of injection pressure on flow length



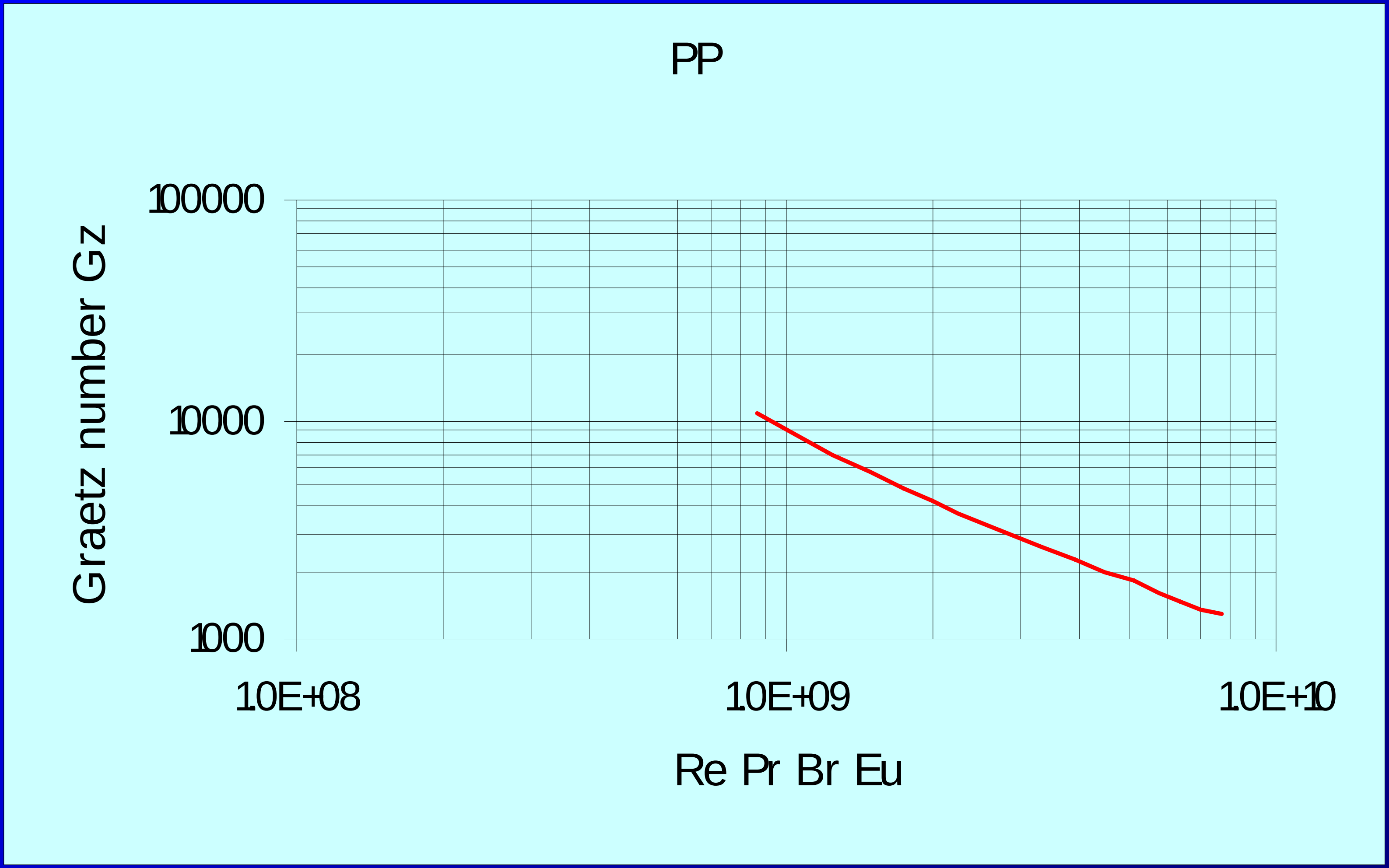
Graetz number as a function of product Re Pr Br Eu



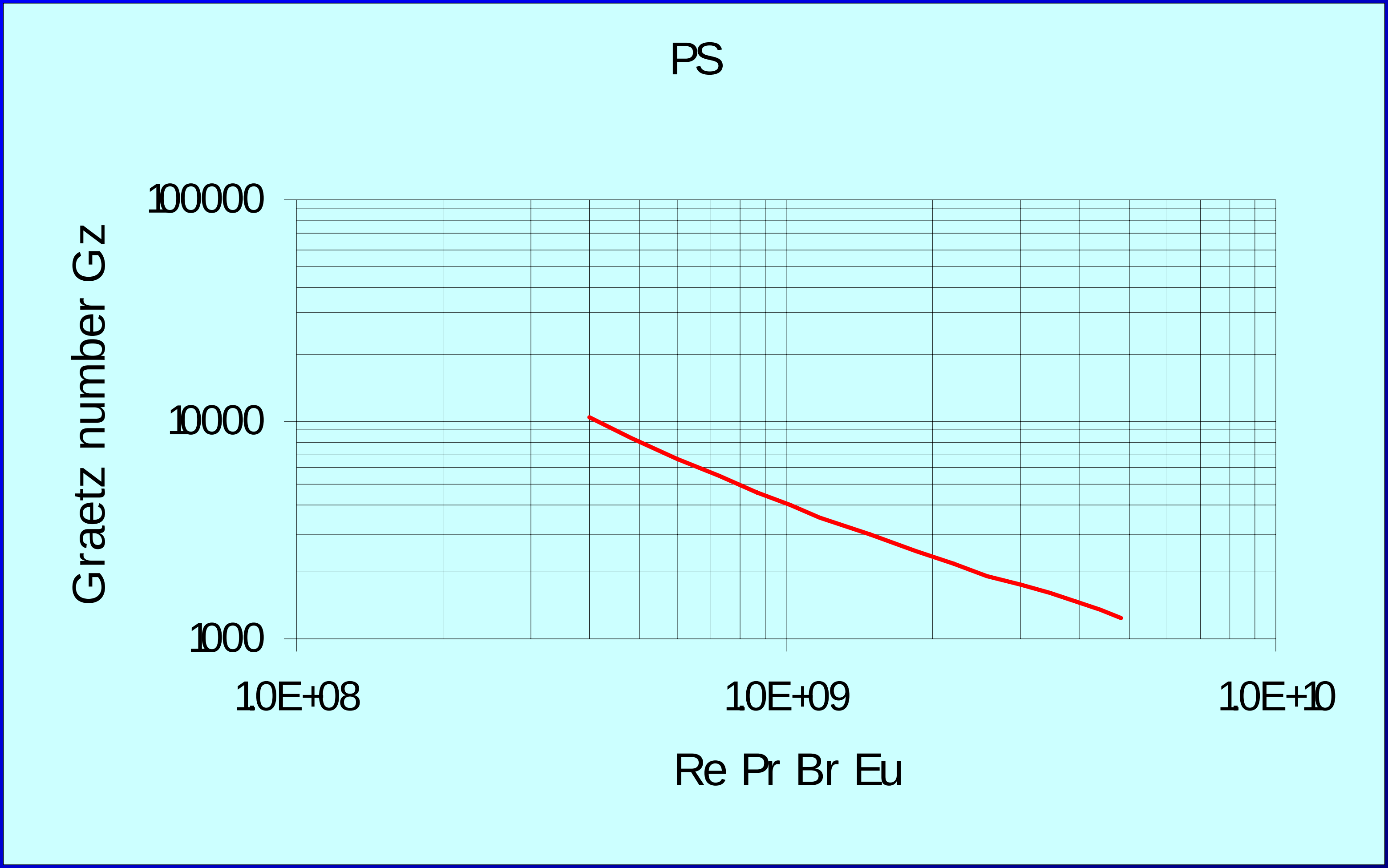
Graetz number as a function of product Re Pr Br Eu



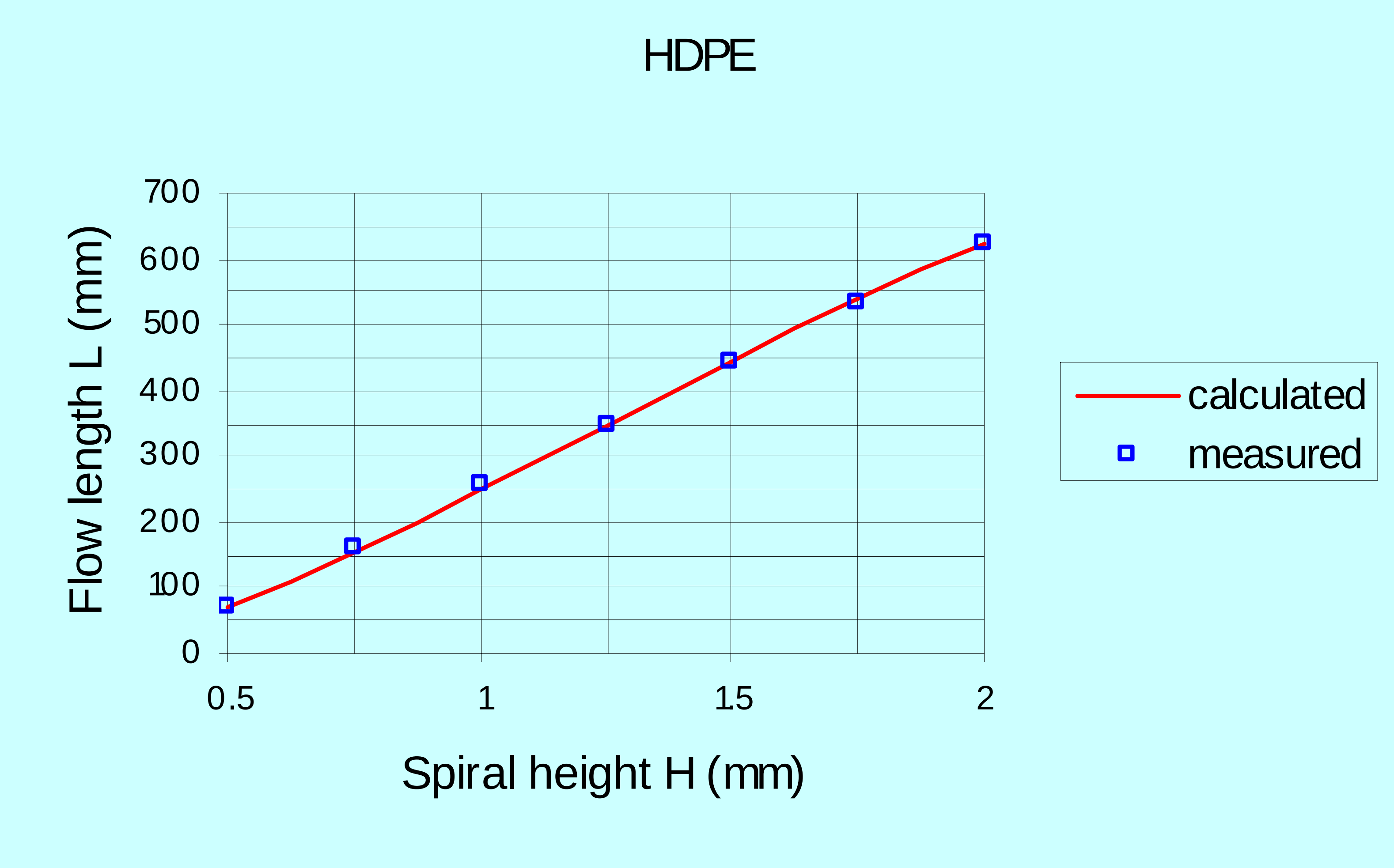
Graetz number as a function of product Re Pr Br Eu



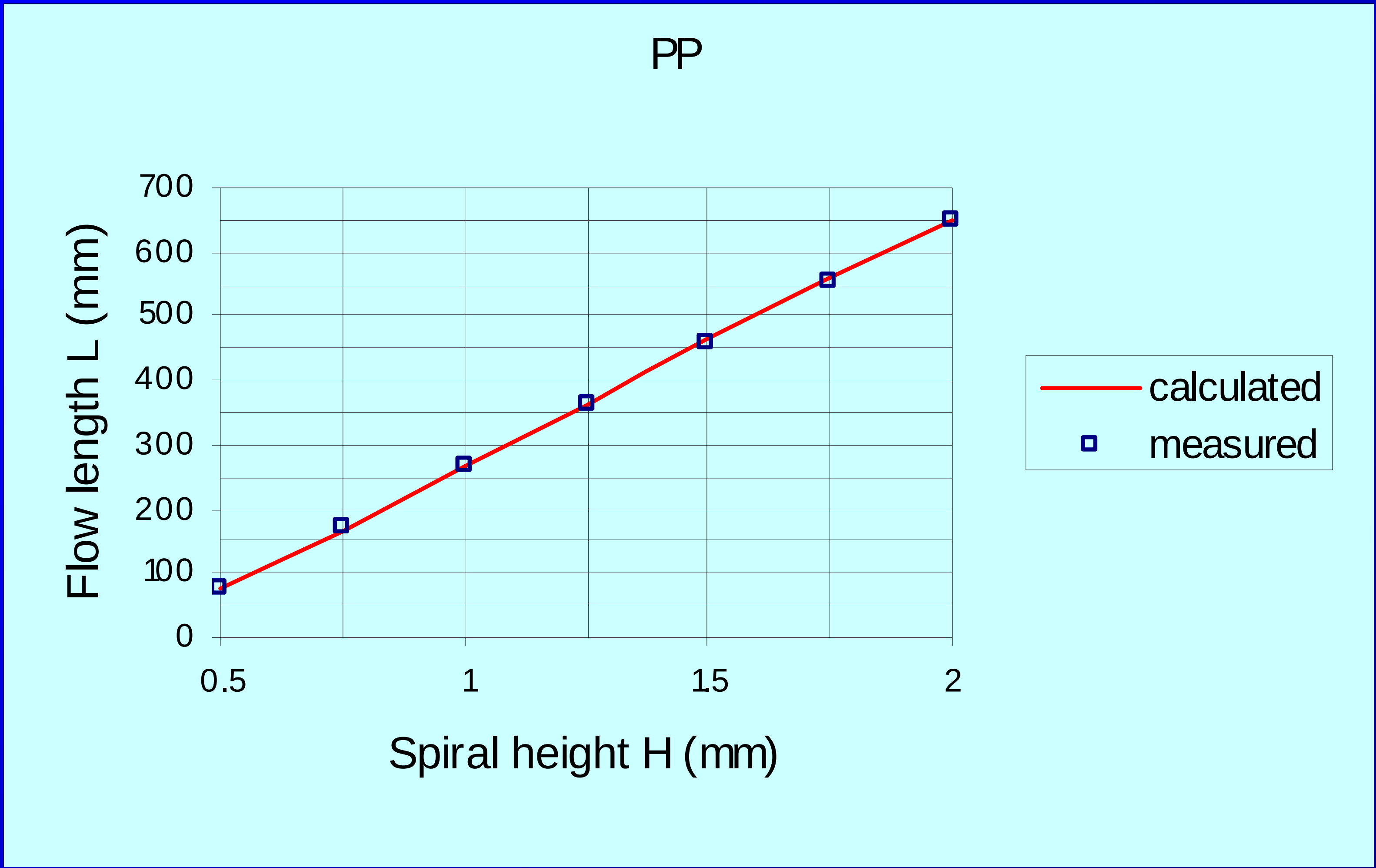
Graetz number as a function of product Re Pr Br Eu



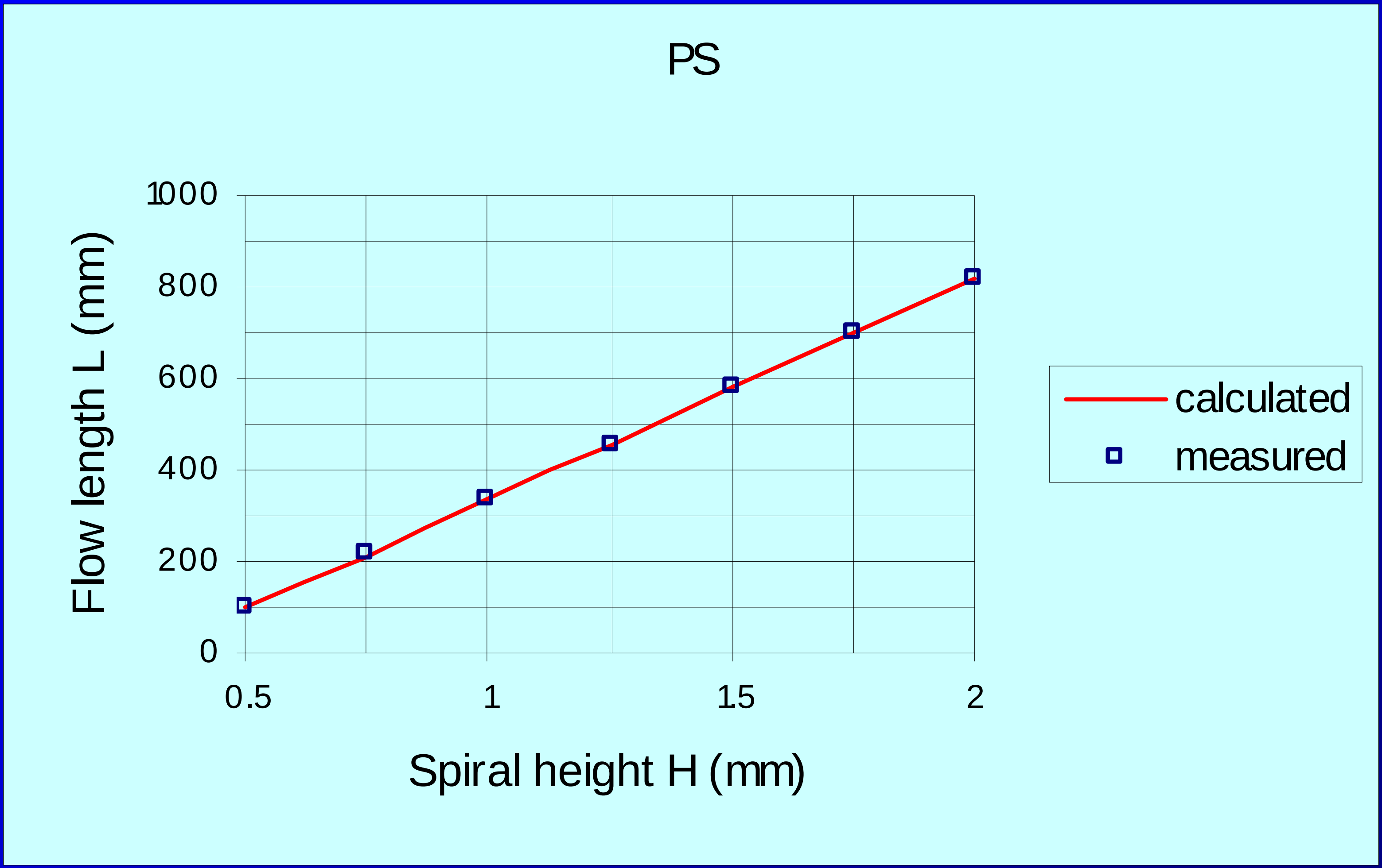
Comparison between measured and calculated flow length



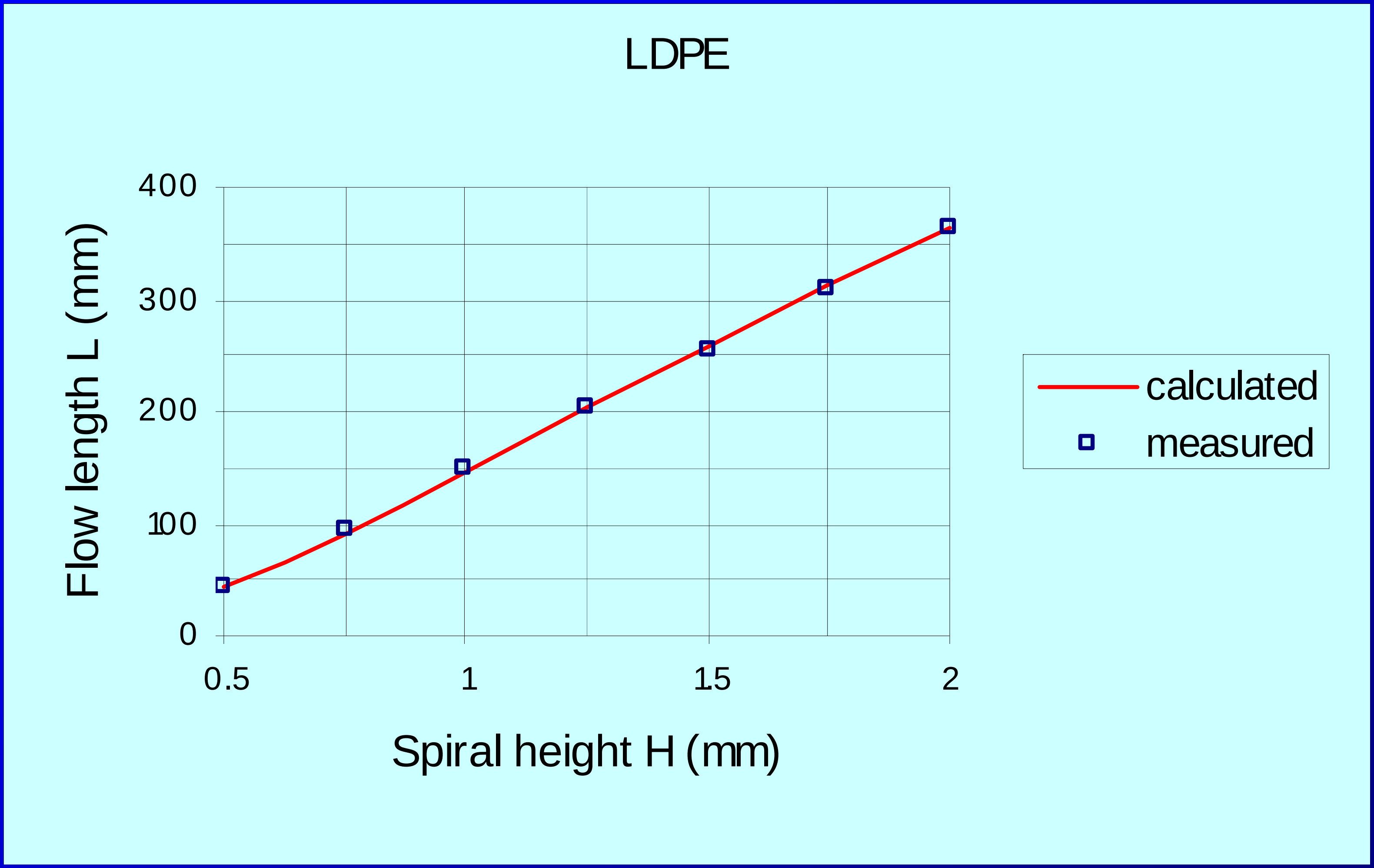
Comparison between measured and calculated flow length



Comparison between measured and calculated flow length

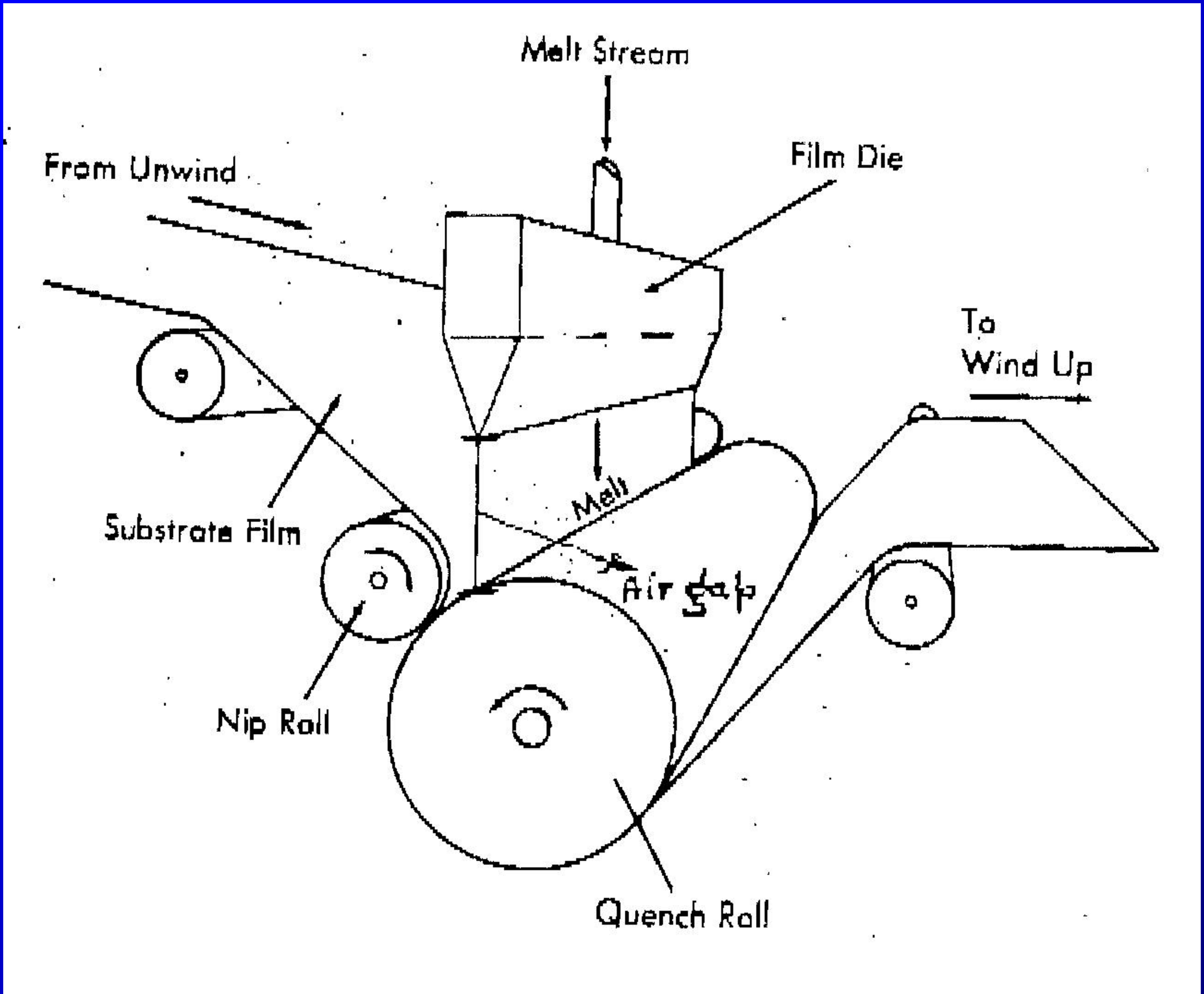


Comparison between measured and calculated flow length



ANALYZING AIR GAP DYNAMICS IN EXTRUSION COATING BY MEANS OF DIMENSIONAL ANALYSIS

-

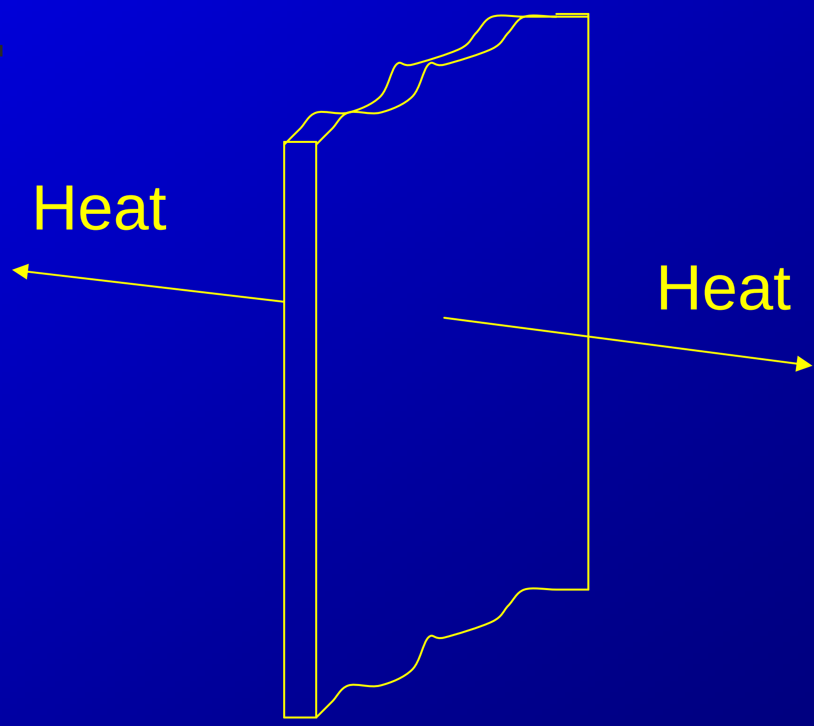

Melt Stream
Film Die
From Unwind
To
Wind Up
Melt
Substrate Film
Air Gap
Nip Roll
Quench Roll

The bond strength is influenced by

- heat transfer
- chemical kinetics and
- rheology

Unsteady-state heat conduction

Heating or cooling of an infinite plate can be calculated by means of Fourier number.



HEAT TRANSFER FROM MOVING FILMS

When the film moves in the air, the heat from the film is controlled by the external resistance.

$$(T - T_\infty)/(T_A - T_\infty) = \exp\left(-\frac{k \cdot A}{\rho \cdot c_p \cdot V} \cdot t\right)$$

T_∞ = air temperature

T_A = initial temperature of the film (melt temperature)

t = time in the air gap

The volume V takes the coat weight into account.

$$s = 1000 \cdot cwt / \rho_{POLYMER}$$

s = coat thickness µ

cwt = coat weight g/m^2

$\rho_{POLYMER}$ = polymer density kg/m^3

For moving films the temperature drop in the air gap can be around 5°C.

CHEMICAL KINETICS
EFFECT OF TEMPERATURE

Effect of temperature

$$a_T = \exp\left[\frac{U}{R}\left(1/T - 1/T_0\right)\right]$$

a_T : shift factor

U : activation energy

R : gas constant

T : melt temperature

T_0 : reference temperature

Graetz Number Gz

$$Gz = \dot{m} \cdot c_p / \lambda \cdot GAPL$$

$\dot{m}$: throughput (kg/h)

c_P : specific heat (kJ/kg K)

λ : thermal conductivity (W/m K)

GAPL : Length of air gap (m)

RHEOLGY

SHEAR HISTORY OF THE FILM

SHEAR =

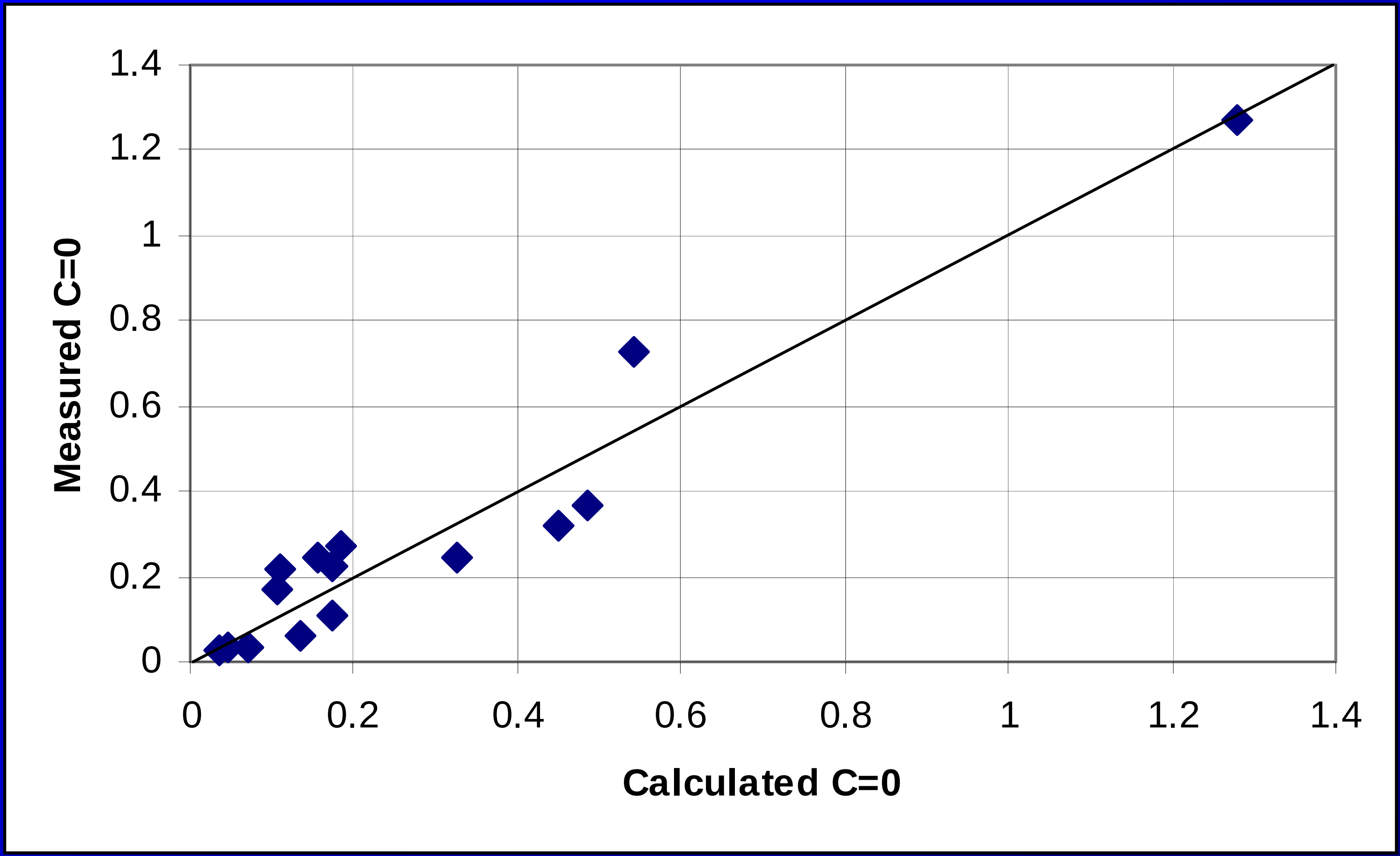
Shear rate at the die exist x TIAG

Evaluation of the Experiments of Ristey and Schroff

Ristey & Schroff - 1978						
Resin: Polyethylene - 5.5 MFR, 0.918 density						
Point #	Melt Temp	Line Speed	Coat Wt	Air-Gap	Air Gap	C=0 abs
	deg C (deg F)	mpm (fpm)	gsm (#/rpm)	mm (inch)	msec	@1720 cm-1
1	321 (610)	274 (900)	12 (7.2)	102 (4)	22	0.026
2	321 (610)	274 (900)	12 (7.2)	203 (8)	44	0.062
3	321 (610)	137 (450)	24 (14.4)	102 (4)	44	0.108
4	321 (610)	91 (300)	24 (14.4)	102 (4)	67	0.172
5	321 (610)	137 (450)	24 (14.4)	203 (8)	89	0.243
6	321 (610)	91 (300)	36 (21.6)	203 (8)	133	0.725
7	329 (625)	274 (900)	12 (7.2)	102 (4)	22	0.035
8	329 (625)	137 (450)	24 (14.4)	102 (4)	44	0.216
9	329 (625)	91 (300)	24 (14.4)	102 (4)	67	0.245
10	329 (625)	91 (300)	36 (21.6)	102 (4)	67	0.322
11	329 (625)	137 (450)	24 (14.4)	203 (8)	89	0.365
12	338 (640)	274 (900)	12 (7.2)	102 (4)	22	0.036
13	338 (640)	137 (450)	24 (14.4)	102 (4)	44	0.227
14	338 (640)	91 (300)	12 (7.2)	102 (4)	67	0.271
15	338 (640)	91 (300)	36 (21.6)	203 (8)	133	1.274

Final Equation

$$\ln(C=0) = -6.7684 - 3.1784\ \ln(\text{shift factor}) - 0.8487\ \ln(Gz) + 1.2784\ \ln(\text{shear})$$



Conclusions

The spiral test is used for classifying injection molding resins with respect to their flowability. Using dimensional analysis and measured flow curves an easily applicable relationship to calculate the spiral flow length was developed. With the help of this equation flow lengths can be estimated thereby minimizing experimentation.

The bonding of laminates in extrusion coating processes served as another example for the application of dimensionless numbers . Using Graetz number the effect of air gap on the adhesion of the laminates is quantified in the form of an equation taking chemical kinetics and rheology into account.

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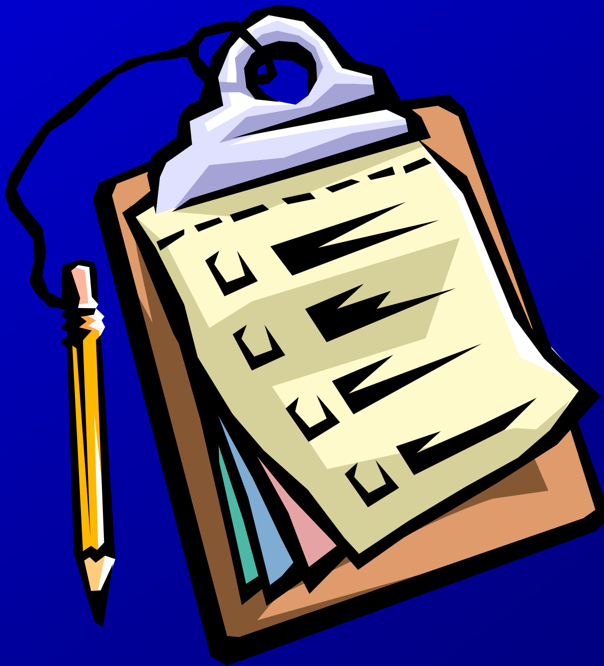

Please remember to turn in your evaluation sheet...