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T 1215

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SUBJECT CATEGORY Optical Properties

RELATED METHODS See "Additional Information"

The determination of instrumental color differences

(Five-year review of Standard Practice T 1215 sp-12)

(Changes from Draft 2 incorporated)

1. Scope

This standard practice provides a general introduction to the use of color differences and a list of the most widely used equations to obtain them. Color differences can be used 1) as a guide to establishing color tolerances in the production of pulp, paper, and paperboard, 2) for the determination of buying and selling tolerances of color, 3) to provide a method of determining the adequacy of color matches.

2. Significance

Although the eye is unsurpassed as a detector of a difference in color between two samples, it is, however, very poor in assessing the size and nature of that difference. Two common examples will attest to this statement: The redder yellow of a pair of yellows will generally be assessed as the stronger one; the brighter *of a* pair of colors will generally be assessed as weaker. This is no fault of the individual, but rather a defect of the visual process, for human vision is not designed to distinguish between the various attributes of color (hue, saturation and lightness) but rather to give an assessment based on an integration of all three. In addition, many other factors influence the visual process: health, lighting, age, etc.

3. Safety precautions

There are no specific safety precautions associated with this standard practice.

4. Development

4.1 Around 1930 several developments laid the foundation for most of the important developments we have had in color instrumentation to date. Ostwald, Munsell, the CIE, and Adams all described color spaces which would help to attach a number designation to a color. Further, the Ostwald, Munsell, and Adams systems attempted to space the colors equally in all directions. The CIE set up the specifications for the Tristimulus Values X, Y and Z in 1931. All data were laboriously hand calculated which was far from a simple routine.

4.2 Several others collaborated during the 1930s and 1940s to develop colorimeters to measure tristimulus values or some modification of them directly. These led to the widely used filter colorimeters.

4.3 Many papers were published which were aimed at giving a more uniform color space in the 1940's. The color difference formulae of McAdam, Adams, Saunderson-Milner, etc. were developed, but the calculations were still too long to be done practically on a production level. In the 1950s came the computational aids: nomographs, tables, charts, and even slide rules. Nickerson (*1*) and Buc (*2*) published tables of data to assist in the computation of the Adams color difference equation. Davidson & Hanlon (*3*), and Simon & Goodwin (*4*) put out charts for calculating color differences according to the MacAdam formula (*5, 6, 7*). Adams, et al (*8*), produced a set of nomographs to simplify the Adams calculations. The introduction of the CODIC (**C**olor **D**ifference **C**omputer), an analog computer, made the calculation in MacAdam's system practical.

4.4 Digital computers became generally available in the 1970's and for the first time in almost 40 years, calculations were no longer an obstacle. Most systems could be readily programmed and computed results from the various formulae compared. MacAdam's system which was first to be rendered usable in a routine manner was simply computerized because constants must be determined from a graph. He published a new procedure for use on a computer. Several modifications were made to give closer agreement to answers received from the charts. In 1964, the CIE adopted a new color space called UCS, and with it an easily programmed color difference formula. This, however, was superseded by the 1976 CIE $u' v'$ system. The Adams equation required Munsell values which are found in tables using the tristimus values, or by computer using the Bridgeman Inversion (*9*). In 1976 the $L^*a^*b^*$ (CIELAB) equation superseded the AN-40 equation using a simple cube root equation. The resulting *E* values from the CIELAB equation are 10% larger than those from AN-40.

4.5 The main purpose of color difference systems is to give a single number value which represents the size of a visual difference between two colors. All of these color difference formulae are composed of chromaticity and of lightness differences. Since many routes were followed to develop a uniform color space (Table 1) and color difference formula, it is not surprising that there is not good correlation between the results of one system and those of any other. A completely uniform space will probably never be developed since it does not conform to Euclidean Geometry.

4.6 The positions of visually equal color differences from a standard were plotted throughout the color space in MacAdam's 1942 paper (*5*). These plots were made on the 1931 CIE chromaticity diagram where the proportions of the tristimulus values are plotted. They showed that the locus of points around a standard is not a circle but rather an ellipse and the major axis is oriented differently in each area of the space. This means, for example, in a green, one can tolerate much more of a saturation difference than a hue difference while the reverse is true of a purple.

5. Procedure

5.1 Reflectance data are obtained for the standard and sample specimens. Two types of instruments are available for this purpose, colorimeters, and spectrophotometers. Colorimeters provide tristimulus data directly such as *X, Y, Z; L, a, b*; or R, G, B. These data are directly substituted in the appropriate color difference equation and the color difference calculated. Manufacturers of the colorimeters may give the conversion formulae needed to convert from one set of reading to another, so that color differences can be computed in other color systems. Some colorimeters have been internally programmed to read out color differences directly. Some instruments can provide color differences in more than one system, illuminant, and standard observer.

5.2 Spectrophotometers provide percent reflectance data at discreet intervals throughout the visible spectrum. The spectral range is commonly 400 to 700 nanometers (nm) at 10 or 20 nanometer intervals. The range may be expanded to include 380 to 740 nm, and the interval may be only 5 nm. These data must be converted to tristimulus values (*10, 11, 12*) before color differences can be calculated. A programmed digital computer is required for this operation. The tristimulus data are then used to compute the color difference between specimens as was described for colorimeters.

6. Official recommendations

6.1 No one color difference formula has been shown to be superior so as to receive the official endorsement of a major standardization body. The CIE (*12, 13,*) recommended the study of four (1964 CIE, Cube-

Root, Munsell Renotation, and the FMC II) with a view toward adoption as an international standard. The Helmholtz Memorial Symposium on Color Metrics reported the disappointing results of seven years of the study (*14*). Two new equations were set forth for the possible adoption (CIE $L^*U^*V^*$. and CIE $L^*a^*b^*$) in 1976 (*16*). Additional terms were defined for hue and saturation.

6.2 The Textile Committee of the International Standards Organization (*16*) suggested the Adams-Nickerson equation be used until a better one is found. They recommended that the factor of 40, as recommended by Nickerson (*1*), be used and the color difference unit be designated AN-40. This unit is commonly, but erroneously, referred to as the NBS unit. The American Association of Textile Chemists & Colorists (AATCC) has used this formula for many years. Both organizations have since recommended the CIELAB system with metric hue angle and chroma because of the similarity with AN-40 results, and its simplicity of calculation. AATCC have more recently adopted the CMC color difference formula as an official test method (*17*).

7. Color difference equations

7.1 The following list contains the better known and more widely used color difference equations. Detailed information about them can be found in the references given. The references also include surveys and comparisons of many of the formulae (*18, 19*). Some recommended practices are given in ASTM-D-2244 (*20*) for several equations.

7.2 In the following list, X, Y, Z refer to Tristimulus values and x, y to the chromaticity coordinates defined as:

$$x = X/(X + Y + Z)$$
$$y = Y/(X + Y + X)$$

7.3 The formulae shown in section 8 are limited to those which are currently in use in the pulp and paper industry. Those equations which predate 1975 and are not currently in use, are considered historical and are not included in this method.

8. Color difference formulae

8.1 Hunter (*27, 28*) Hunter L, a_L, b_L system

$$\Delta E\ [(\Delta L)^2 + (\Delta a_L)^2 + (\Delta b_L)^2]^{0.5}$$

where:

$L = 10Y^{0.5}$

$a_L = 17.5(1.02X - Y)/Y^{0.5}$

$b_L = 7.0(Y - 0.847Z)/Y^{0.5}$

8.2 CIE 1976 ($L^*u^*v^*$)

$\Delta E = [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{1/2}$

where:

$L^* = 25(100Y/Y_0)^{1/3} - 16; 1 \leq Y \leq 100$

$u^* = 13\, L^*(u' - u'_0)$

$v^* = 13\, L^*(v' - v'_0)$

with:

$$u' = \frac{4X}{X + 15Y + 3Z} \qquad v' = \frac{9Y}{X + 15Y + 3Z}$$

$$u'_0 = \frac{4X}{X_0 + 15Y_0 + 3Z_0} \qquad V'_0 = \frac{9Y_0}{X_0 + 15Y_0 + 3Z_0}$$

X_0, Y_0, Z_0 define the color of the nominally white object-color stimulus, with $Y_0 = 100$

8.3 CIE 1976 ($L^*a^*b^*$), CIELAB (*21*), Pauli (*22*)

$\Delta E = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2}$

where:

$L^* = 116(Y/Y_n)^{1/3} - 16$

$a^* = 500[(X/X_n)^{1/3} - (Y/Y_n)^{1/3}]$

$b^* = 200[(Y/Y_n)^{1/3} - (Z/Z_n)^{1/3}$

X/X_n, Y/Y_n, $Z/Z_n > 0.01$

The tristimulus values X_n, Y_n, Z_n define the color of the nominally white object-color stimulus. Usually, the white object-color stimulus is given by the spectral radiant power of one of the CIE standard illuminants reflected into the observer's eye by the perfect reflecting diffuser. Under these conditions, X_n, Y_n, Z_n are the tristimulus values of the standard illuminant with Y_n equal to 100.

The total difference ΔE^*_{ab} between two colors each given in terms of L^*, a^*, b^* is calculated from:

$$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2}$$

In calculating L^*, values of Y/Y_n less than 0.01 may be included if the normal formula is used for values of Y/Y_n greater than 0.008856, and the following modified formula is used for values of Y/Y_n equal to or less than 0.008856:

$$L^* = 903.3(Y/Y_n) \qquad Y/Y_n \leq 0.008856$$

In calculating a^* and b^*, values of X/X_n, Y/Y_n, Z/Z_n less than 0.01 may be included if the normal formula are replaced by the following modified formulae:

$$a^* = 500[\int(X/X_n) - \int(Y/Y_n)]$$
$$b^* - 200[\int (Y/Y_n) - \int(Z/Z_n)]$$

where:

$$\int (X/X_n) = (X/X_n)^{1/3} \qquad X/X_n > 0.008856$$
$$\int (X/X_n) = 7.787(X/X_n) + 16/116 \qquad X/X_n < 0.008856$$
$$\int (Y/Y_n) = (Y/Y_n)^{1/3} \qquad Y/Y_n > 0.008856$$
$$\int (Y/Y_n) = 7.787(Y/Y_n) + 16/116 \qquad Y/Y_n < 0.008856$$
$$\int (Z/Z_n) = (Z/Z_n)^{1/3} \qquad Z/Z_n > 0.008856$$
$$\int (Z/Z_n) = 7.787(Z/Z_n) + 16/116 \qquad Z/Z_n < 0.008856$$

X_0, Y_0, Z_0 are as defined in CIE 1976 ($L^*u^*v^*$)

This space and formula are simplified versions of the Adams-Nickerson version. It is intended to give a more easily computed space and equation. They are widely used for colorant formulation, color tolerances, and quality control determinations.

8.4 A CMC color difference formula was developed by the Color Measurement Committee of the Society of Dyers and Colorists (*18, 23, 24*). The CMC formula evolves from the CIELAB system, previously described. An improvement is claimed in agreement between calculated color difference values and visual assessment of color difference.

The coordinates lightness (L^*), chroma (C^*), and hue angle (h) are derived from CIE L^*, a^* and b^* values:

$$C^* = [a^{*2} + b^{*2}]^{1/2}$$
$$h = \arctan (b^*/a^*)$$

Color difference calculated in terms of L^*, C^* and h is:

$$\Delta E^* = [\Delta L^{*2} + \Delta C^{*2} + \Delta H^{*2}]^{1/2}$$

where for small color differences

$$\Delta H^* = C^* \Delta h\ (\pi/180)$$

The CMC (ℓ:c) formula is then:

$$\Delta E \text{ CMC } (\ell{:}c) = [\Delta L^*/\ell S_L)^2 + (\Delta C^*/cS_c)^2 + (\Delta H^*/S_H)^2\ ]^{1/2}$$

where:

$$S_L = 0.040975\ L^*/(1 + 0.01765\ L^*) \text{ unless } L^* < 16 \text{ when } S_L = 0.511$$
$$S_C = [0.0638\ C^*/(1 + 0.0131 C^*)] + 0.638$$
$$S_H = S_C\ (FT + 1 - F)$$
$$F = \{(C^*)^4/[(C^*)^4 + 1900]\}^{0.5}$$
$$T = 0.36 + \text{ABS}\ [0.4 \cos\ (h + 35)] \text{ unless } h \text{ is between } 164° \text{ and } 345°$$
$$\text{when } T = 0.56 + \text{ABS}\ [0.2 \cos\ (h + 168)]$$

When $\ell = c = 1$, the formula quantifies the *perceptibility* of color difference. Optimum values for ℓ and c may be determined for quantifying the *acceptability* of a color match.

8.5 CIE ΔE^*_{94}

This is a CIE recommendation on the practice of industrial color difference evaluation (*25*). It does not have the status of a CIE standard. The CIE 1976 ($L^*a^*b^*$) color space (see 8.3) is retained as an approximate uniform color space representing perception color magnitudes in terms of opponent color scales.

The total color difference ΔE^*_{94} between two color samples is a weighted Euclidean distance in CIE 1976 ($L^*a^*b^*$) color space with rectangular color difference components; ΔL^*, ΔC^*_{ab}, ΔH^*_{ab}. A perceived color difference magnitude, ΔV, is related to the total color difference through an overall sensitivity factor, K_E.

$$\Delta V = K_E^{-1}\,\Delta E^*_{94}$$

$$\Delta E^*_{94} = \left[\left(\frac{\Delta L^*}{K_L\ S_L}\right)^2 + \left(\frac{\Delta C^*_{ab}}{K_C\ S_C}\right)^2 + \left(\frac{\Delta H^*_{ab}}{K_H\ S_H}\right)^2\right]^{0.5}$$

Weighting functions S_L, S_C, S_H adjust the total color difference equation to account for variation in perceived color difference magnitude with variation in the color standard location in color space.

$$S_L = 1$$
$$S_C = 1 + 0.045C^*_{ab}$$
$$S_H = 1 + 0.015C^*_{ab}$$

Parametric factors K_L, K_C, K_H are correction terms for variation in perceived color difference component sensitivity with variation in experimental conditions. These parametric factors may be defined by industry groups to correspond to typical experimental conditions for that industry.

The CIE ΔE^*_{94} formula was derived from the CMC formula (see section 8.4), which is widely used in some industries, and is generally acknowledged to give better results than CIELAB for correlation with visual assessment.

8.6 CIE ΔE^*_{00}

This is a CIE recommendation on the practice of industrial color difference evaluation (*26*), and includes improvements to the ΔE^*_{94} color difference equation. It does not have the status of a CIE standard. The CIE 1976 ($L^*a^*b^*$) color space (CIE, 1986) is retained as an approximate uniform color space representing perceptual color magnitudes in terms of opponent color scales with a localized modification to the a* (red-green opponent) axis. This modification was made to improve agreement with visual color-difference perception for neutral colors. The modified color coordinates are given as primed (') quantities and derived by the following equations:

$$L' = L^*$$

$$a' = a^*(1+G) \quad \text{where} \quad G = 0.5\left(1 - \sqrt{\frac{\bar{C}^{*\,7}_{ab}}{\bar{C}^{*\,7}_{ab} + 25^7}}\right)$$

$$b' = b^*$$

The modification increases the magnitudes of a' values compared to a* values for colors at low chroma. At higher chroma the modified a' value approaches the conventional a* value. Primed quantities refer to quantities derived from L', a', b' coordinates. CIE 1976 chroma (C'_{ab}) and CIE1976 hue angle (h'_{ab}) are calculated using the a', b' coordinate values when they are used in color-difference calculations:

$$C' = (a'^2 + b'^2)^{.5}$$

$$h' = tan^{-1}(\frac{b'}{a'})$$

The total color difference ΔE^*_{00} between two color samples is a weighted Euclidean distance in CIE 1976 ($L^*a^*b^*$) color space with rectangular color difference components; ΔL^*, ΔC^*_{ab}, ΔH^*_{ab}. A perceived color difference magnitude, ΔV, is related to the total color difference through an overall sensitivity factor, K_E.

$$\Delta V = K_E^{-1}\,\Delta E^*_{00}$$

$$\Delta E^*_{00} = \left[\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + \left(R_T\left(\frac{\Delta C'}{k_C S_C}\right)\left(\frac{\Delta H'}{k_H S_H}\right)\right)\right]^{.5}$$

color difference components are given by:

$$\Delta L' = L'_b - L'_s$$

$$\Delta C' = C'_b - C'_s$$

$$\Delta H' = 2(C'_b C'_s)^{.5} \sin\left(\frac{\Delta h'}{2}\right)$$

where

$$\Delta h' = h'_b - h'_s$$

Subscripts s and b refer to standard and batch respectively of a color-difference pair.

Weighting functions S_L, S_C, S_H adjust the total color difference equation to account for variation in perceived color difference magnitude with variation in the color standard location in color space.

$$S_L = 1 + \frac{0.015(\overline{L'} - 50)^2}{\sqrt{20 + (\overline{L'} - 50)^2}}$$

$$S_C = 1 + 0.045\overline{C'}$$

$$S_H = 1 + 0.015\bar{C}'T$$

Where

$$T = 1 - 0.17\cos(\bar{h}' - 30) + 0.24\cos(2\bar{h}') + 0.32\cos(3\bar{h}' + 6) - 0.20\cos(4\bar{h}' - 63)$$

Visual color-difference perception data show an interaction between chroma difference and hue difference in the blue region that is observed as a tilt of the major axis of a color-difference ellipsoid from the direction of constant hue angle. To account for this effect a rotation function is applied to weighted hue and chroma differences:

$$R_T = -\sin(2\Delta\theta)R_C$$

$$\Delta\theta = 30\exp\left\{-\left[(\bar{h}' - 275)/25\right]^2\right\}$$

$$R_C = 2\sqrt{\frac{\bar{C}'^7}{\bar{C}'^7 + 25^7}}$$

The parametric factors, kL, kC, kH are correction terms for variation in experimental conditions. Under reference conditions they are all set at 1. For other choices see (CIE, 1993). The reference conditions are:

Illumination: source simulating the spectral relative irradiance of CIE standard illuminant 065.
Illuminance: 1000 Ix
Observer: normal color vision
Background field: uniform, neutral grey with L* = 50
Viewing mode: object
Sample size: greater than 4 degrees subtended visual angle
Sample separation: minimum sample separation achieved by placing the sample pair in direct edge contact
Sample color difference magnitude: 0 to 5 CIELAB units
Sample structure: homogeneous color without visually apparent pattern or non-uniformity

9. Keywords

Color, Optical measurement, Optical instruments

10. Additional information

10.1 Effective date of issue: to be assigned.

10.2 This standard practice was formerly TIP 0804-04 and was revised in 1998. The 2007 version included only editorial changes. In the 2012 version, the equation in 8.5 was corrected.

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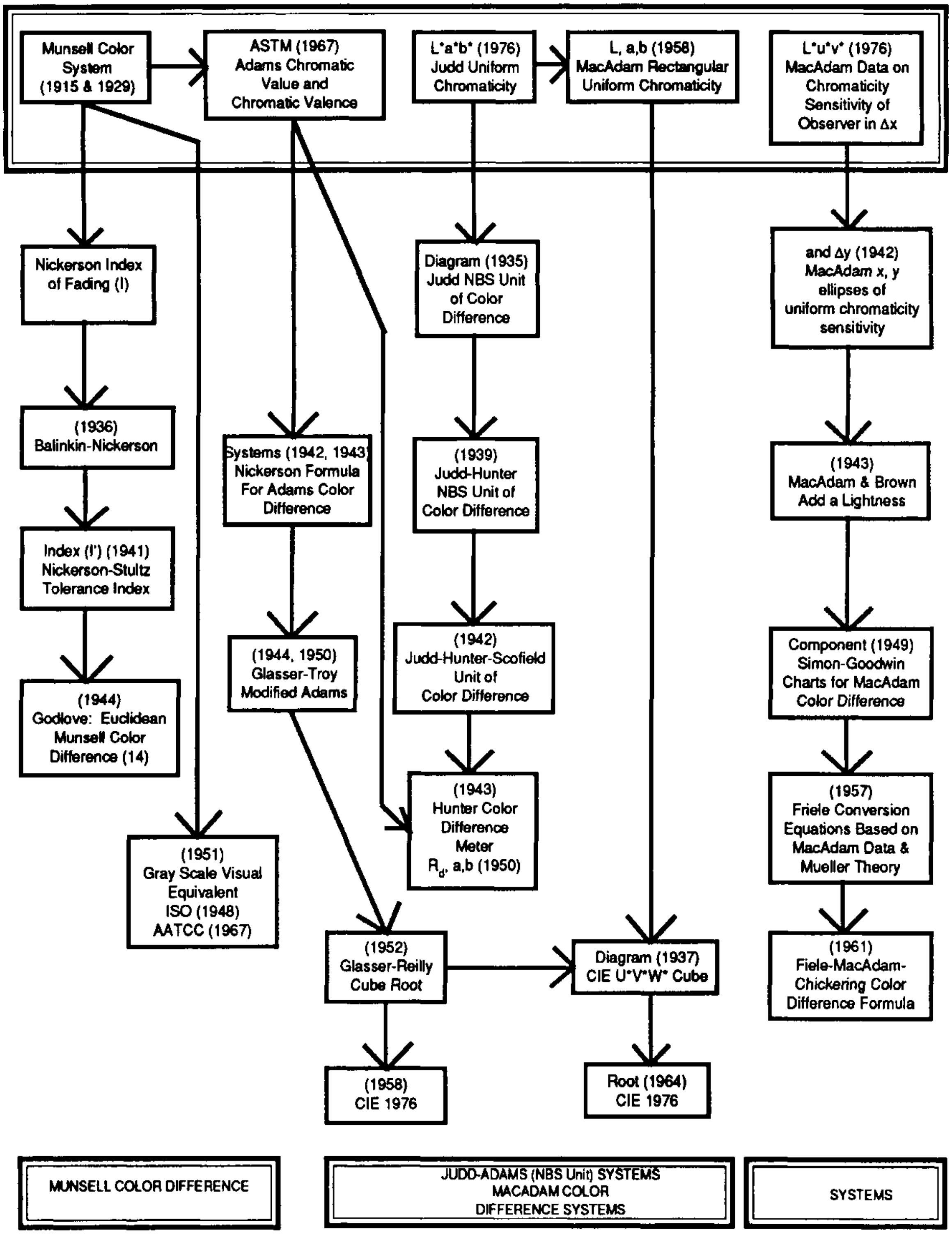
Key to abbreviations of journals

ADR	American Dyestuff Reporter
AIC	International Colo(u)r Association
AO	Applied Optics
CE	Color Engineering
CIE	International Committee on Illumination
ISO	International Organization for Standardization
JOSA	Journal of the Optical Society of America
OF DIG	Official Digest
PTJ	Paper Trade Journal
TEX R	Textile Research

Table 1. Major color difference systems.



Your comments and suggestions on this procedure are earnestly requested and should be sent to the TAPPI Standards Department. ■